Estimating Phase Maps from Partial K Space Data

Introduction Estimation of phase maps is an important step in many phase-contrast MRI applications. Information about numerous physical properties such as B_0 field [1], chemical shift effects [2], flow or motion [3], temperature [4], and even tissue elasticity [5] can be encoded in the phase of MRI signal. Typically, a phase map is obtained by direct comparison of two complex images acquired with different phase sensitivities for the physical quantity of interest. This approach is a pixel-by-pixel computation and is sensitive to local noise, particularly in regions of low signal. Moreover, this method generally requires at least two complete scans to obtain the required phase information over the full field-of-view (FOV). While reduction of scan time can be achieved by using a central portion of *k*-space if the phase is known to be smoothly varying, truncation artifacts can adversely affect the subsequent phase map estimation.

We present an alternative method that offers robust estimation of phase maps with minimal required input data and low sensitivity to local noise. The principle draws directly from the convolution theorem of the Fourier Transform (FT); a multiplication in image space corresponds to a convolution in *k*-space [6]. If two image domain datasets differ by a phase factor, $\exp[i\varphi(x,y)]$, then there is a corresponding convolution kernel $h(k_x,k_y) = \mathrm{FT}^{-1}\{\exp[i\varphi(x,y)]\}$ relating the *k*-space signals. The kernel h, can be determined by a fitting process similar to that used in GRAPPA [7] and ORACLE [8], even if only a small fraction of *k*-space data is available. In this study, we investigate the effectiveness of phase map estimation related to various *k*-space coverage patterns and convolution kernel sizes. The fitted results were compared with a gold-standard phase map obtained from two complex images generated with full *k*-space data.

Methods Two 256x256 complex images were generated by imposing random but identical phase to a Shepp-Logan image. An additional phase modulation of the form $\exp[i\varphi_0(x,y)]$ was applied to one of the simulated images, creating a known relative phase difference between them. The corresponding simulated *k*-space data, f_1 and f_2, were obtained from the FT of the images. Estimated relative phase maps were generated using the pixel-by-pixel conjugate product method [9], as well as via convolution fitting with different samples of the available *k*-space data. Each technique provides a phasor representing the phase difference map that is obtained with a particular *k*-space coverage, f_1' and f_2'. For the *k*-space fitting, $P_h = \exp[i\varphi_h(x,y)]$, is computed from the IFT of the kernel, h. For the conjugate product method, the phasor $P_t = \exp[i\varphi_t(x,y)]$, is obtained from $C_t = \mathrm{FT}^{-1}\{f_1'\}(\mathrm{FT}^{-1}\{f_2'\})^*$, the image space conjugate product from the truncated *k*-space. The phasors found with the sample data are compared with a gold-standard phasor $P_0 = \exp[i\varphi_0(x,y)]$, associated with the conjugate product using full *k*-space data, C_0. The agreement between phase maps is described by a phase refocusing ratio (RR) similar to the one employed in [10] (see Eqn. 1 in Fig. 2a). A RR of unity represents perfect agreement between the estimated phase map and the gold-standard.

Results The gold-standard map in Fig. 1a displays the target phase map between images, $\varphi_0(x,y)$, which is parabolic in this case. Fig. 1b shows the estimated phase map, $\varphi_t(x,y)$, computed from 20 central *k*-space lines (Fig. 1e is the magnitude of the complex difference map between 1a and 1b). Both the estimated phase map and the complex difference map show the effects of truncation artifact. Fig. 1c displays the phase map generated by fitting an 8x8 kernel to the same 20 *k*-space lines, (Fig. 1f, the corresponding difference map). The resulting phase map is smoother and does not suffer from truncation effects. The kernel fitting technique provides a more accurate estimation of relative phase than the image space technique when dealing with limited sample data. The robustness of the fitting is further displayed in Fig. 2; if an adequate kernel size is chosen (Fig. 2a), the fitting performs better than the conjugate phase method when using highly under-sampled data (Fig. 2b). The data may be sampled from various regions of *k*-space, however for optimal results the coverage should be near central *k*-space (Fig. 2c).

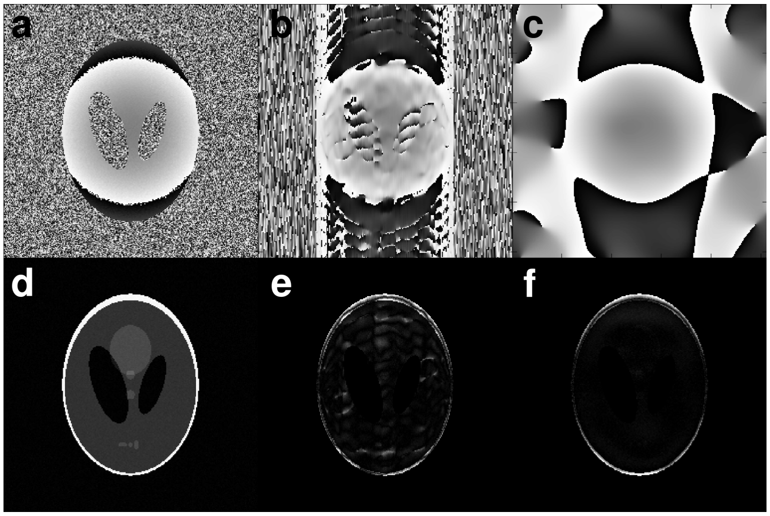

Figure 1 **(a)** Gold-standard (parabolic) phase difference map, **(b)** conjugate product phase map from truncated *k*-space (20 k_y lines), **(c)** convolution fitting phase map (8x8 kernel, 20 k_y lines), **(d)** Magnitude image displaying important signal region, **(e)** and **(f)** difference maps of the phases shown in (b) and (c) relative to (a), respectively.

Discussion Estimation of relative phase maps is achievable via *k*-space convolution fitting. The fitted maps are smooth and are not sensitive to noise or truncation artifact. The results are comparable with current standards and actually prove superior when dealing with limited *k*-space data. This technique provides an alternative method for estimating phase maps that needs less scan time than prevalent image domain techniques requiring full FOV acquisitions.

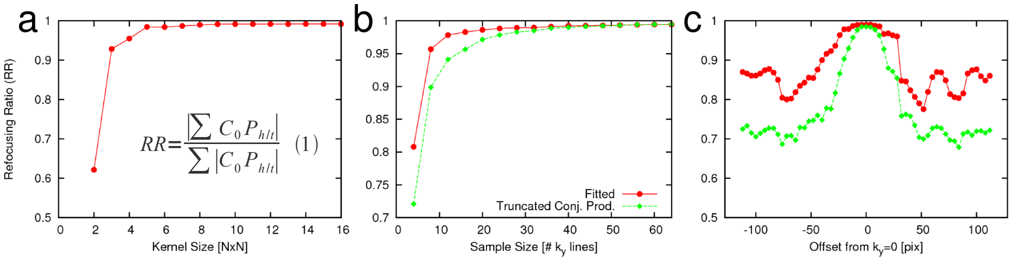

Figure 2 Refocusing ratio of estimated phase maps versus (**a**) kernel size (sample size 32 k_y lines), (**b**) sample size (kernel size 8x8) and **(c)** offset from central *k*-space (8x8 kernel, 32 k_y lines).

References

[1] P Jezzard & RS Balaban, MRM, 34:65-73, 1995
[2] WT Dixon, Radiology, 153:189-194, 1984
[3] PR Moran, MRI, 1:197-203, 1982
[4] Y Ishihara *et al*., MRM, 34:814-823, 1995
[5] R Muthupillai *et al.*, MRM, 36:226-274, 1995
[6] RN Bracewell, The Fourier Transform and its Applications, McGraw-Hill, 1965
[7] M Griswold *et al*., MRM, 47:1202-1210, 2002
[8] W Lin *et al.*, ISMRM Proceedings 19, 2011
[9] QS Xiang & FQ Ye, MRM, 57:731-741, 2007
[10] Z Chang & QS Xiang, IEEE Trans. Med. Imag., 24:791-798, 2005