Rician Noise Removal in Diffusion Kurtosis Imaging

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Introduction: In the past decade, many denoising algorithms have been proposed for MR images. Most often the Gaussian filter is used for this purpose. However it has already been proven that noise on the magnitude MR images is not Gaussian, but Rician distributed (1). Gaussian filters are used in general for the reason that the noise distribution of magnitude MR images with high signal-to-noise ratio (SNR) can be well approximated with a Gaussian distribution. Diffusion Kurtosis Imaging (DKI), an extension of Diffusion Tensor Imaging (DTI), generates images with a lower SNR compared to images obtained with DTI due to the need of larger and more b-values (2). Subsequently the noise distribution in obtained DKI MR images can no longer be treated as Gaussian distributed. In this study, two Rician filters, namely the non-local mean Rician filter (NLM) and the non-local maximum likelihood Rician filter (NLML), and Gaussian filter (G) are investigated on their denoising performance on DKI MR images. Results obtained from the simulations show more accurate derived parameters from DKI images denoised with the Rician filters than those from images denoised with the Gaussian filter. In addition, regarding the parameters' value derived from real human data, there are significant differences between those from data filtered with the Rician filters and those from data filtered with Gaussian filter.

Algorithm: The NLM filter is an adaption of the non-local mean filter to denoise Rician noise in MR images (3). It denoises a pixel y according to the formula [1], where x_i is one of a series of its surrounding pixels which are not in the direct neighborhood of y; w_i is the normalized weight of x_i. w_i is derived by evaluating the similarity between the direct neighborhood of the y and those neighborhood pixels of x_i. The NLML filter also denoises the pixels in a non-local way but the noise free value Y is more accurately estimated based on a likelihood function instead of simply taking the weighted average value of a series of surrounding neighborhood pixels (4). According to formula [2] and [3], the NLML filter estimates the noise free value of pixel y by maximizing the log-likelihood function log L with respect to Y. Both NLM filter and NLML filter are designed to perform on MR images with low SNR.

$$\mathrm{NLM}(y) = \mathrm{sqrt}\left(\left(\sum \mathrm{w_i x_i^2}\right) - 2\sigma^2\right) \quad [1] \qquad \mathrm{NLML}(y) = \arg\{\max_{\mathrm{Y}}(\log \mathrm{L})\} \quad [2] \qquad \log \mathrm{L} = \sum \log\left(\frac{\mathrm{x_i}}{\sigma^2}\right) - \sum \frac{\mathrm{x_i + Y^2}}{2\sigma^2} + \sum \log \mathrm{I_0}\left(\frac{\mathrm{Yx_i}}{\sigma^2}\right) \quad [3]$$

After denoising, the parameters of fractional anisotropy (FA), mean diffusivity (MD), mean kurtosis (MK), Kaxial ($K_\parallel$), Kradial ($K_\perp$), Daxial ($D_\parallel$) and Dradial ($D_\perp$) are calculated from the denoised images according to the model described in formula [4]. All parameters are derived using in-house own written MATLAB (Mathworks, Natick, MA, USA) scripts based on the convex quadratic program method (5).

$$\ln\left(\frac{S(g,b)}{S_0}\right) = -bD(g) + \frac{1}{6}b^2D^2(g)K(g) \quad [4]$$

Experiment: Synthetic data: The synthetic image is made based on a diffusion weighted data set generated with the three different b-values and 32 gradient directions. First several coordinates in the b_0 image are selected, and then all the intensities that correspond with the selected coordinates in all the images are used to create the synthetic image. Monte Carlo simulations (n=100) were done to explore the difference between the Rician filters and the Gaussian filter when applied on an artificially corrupted image for denoising. This synthetic image is considered to be noise-free and Rician noise is added for every noise free pixel x following the formula [5]:

$$x_r = \sqrt{\left(\frac{x}{\sqrt{2}} + n_r\right)^2 + \left(\frac{x}{\sqrt{2}} + n_i\right)^2} \quad [5] \qquad PE = \frac{|P_{nf} - P_d|}{P_{nf}} \quad [6] \qquad PD = \frac{|P_{on} - P_d|}{P_{on}} \quad [7]$$

where x_r is the corrupted pixel x with Rician noise, n_r and n_i the real and imaginary noise having a Gaussian distribution N $(0,\sigma^2)$. By varying σ the range of SNR is 4 ~ 16. The corrupted images are then denoised once with the Rician filters (with the same window size) and once with a Gaussian filter (FWHM = 1.25 pixels). Each parameter derived from the three different denoised and filtered images is compared with the corresponding parameter derived from the noise-free image by calculating the average value of the percentage error (PE). The PE is defined as [6], where P_{nf} is the noise-free parameter value and P_d the parameter value derived from the denoised image. For all calculations and image denoising, an in-house own written Matlab script is used. Figure 1 shows the results of the simulations. Rician filters, especially NLML, outperform Gaussian filter for illustrated parameters in the SNR range of 5 ~16, while NLML outperforms NLM for FA, MD and MK especially in low SNR range of 5 ~ 10.

Real brain data: The performances of the filters were validated on real human data (n = 15). The scan for the human data was performed on a Philips 3T MRI Achieva scanner (Philips Healthcare, Best, The Netherlands) with a single-shot EPI sequence and 3 different b-values (0, 1000 and 2000 s/mm^2). TR/TE=2000/69 ms, reconstruction resolution = 2 x 2 x 3 mm^3. Percentage difference (PD) was defined as [7], where P_{on} is the parameters' value from original image with noise, and P_d is the parameters' value from denoising filtered images. The results of the test on real human data (Table 1) show that there are significant differences in parameter values between the Gaussian filter and the Rician filters, and between NLM filter and NLML filter.

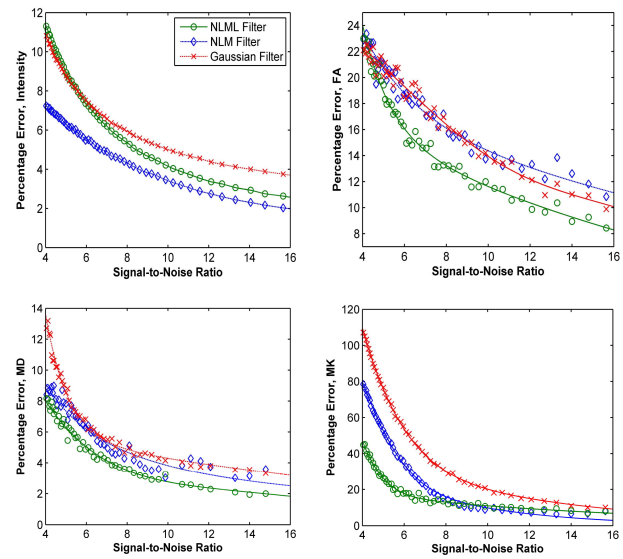

Figure 1. The SNR plotted against the percentage error for (from up left to down right): the intensity, FA, MD and MK.

Para-meters	PD of G		PD of NLM		PD of NLML		G vs NLM	G vs NLML	NLM vs NLML
	Mean	Std	Mean	Std	Mean	Std	P value	P value	P value
FA	31.45	1.74	7.81	5.87	18.59	4.29	<0.001	<0.001	<0.001
MD	16.90	1.84	2.39	3.36	4.72	3.54	<0.001	<0.001	0.005
MK	16.18	1.27	4.13	4.46	9.15	4.15	<0.001	<0.001	0.003
$D_\parallel$	16.15	1.39	2.50	3.31	5.66	3.24	<0.001	<0.001	0.004
$D_\perp$	20.18	1.70	2.95	3.85	5.69	3.99	<0.001	<0.001	0.004
$K_\parallel$	25.19	4.19	8.66	6.48	20.55	4.71	<0.001	0.010	<0.001
$K_\perp$	26.12	4.54	7.61	6.04	17.85	5.17	<0.001	<0.001	<0.001

Table 1. Percentage difference of parameters and the results of Mann-Whitney U test between different filters.

Discussion and Conclusions: The simulation shows that the NLML filter outperforms the Gaussian filter in parameters estimation for the whole SNR range, especially for low SNR range of 5 ~ 10. While NLM filter outstrips Gaussian filter only in MD and MK. A possible explanation is that the NLML takes the log likelihood estimation rather than simply averaging the weighted neighborhood pixels' value, so it performs superior to NLM in parameters' accuracy. Further validating test on real human data shows that using Rician filters (NLML and NLM) could possibly lead to significant difference for parameters estimation compared to generally choosing a Gaussian filter.

References: [1] M. A. Bernstein et al., Med. Phys. 1989, 15(5), 813-817. [2] Lu H et al., NMR Biomed. 2006; 19(2): 236–247. [3] Wiest-Daesslé N et al., LNCS 5242, 2008;11(Pt 2):171-9. [4] L. He et al., IEEE TMI 2009, 28(2):165-72. [5] Tabesh A. et al., Magn Reson Med. 2011, 65(3):823-36.