

# Distortion correction using the susceptibility based field map estimation in echo planar imaging reconstruction

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## Introduction

The echo planar imaging (EPI) technique plays an important role in functional MRI because of its high temporal resolution, that enables us to accurately analyze in-vivo activities related to the brain functionality. However, reconstructed EPI images suffer from geometric distortion often due to the magnetic field inhomogeneity and significantly undermine the performance of activity analyses. Save for a few special acquisition sequences, an accurate field map for each EPI slice is unavailable and a predetermined static field map may change due to the prevalent subject motion during scanning. Common approaches to the image distortion correction include unwarping in the spatial domain by shifting the reconstructed voxels or resampling k-space data using the estimated field map. In this work, we focus on a fundamental approach on the construction of the susceptibility-induced magnetic field inhomogeneity map, that is necessary to recover the spin density for EPI reconstruction. By modeling the acquisition process, we identify the effect of the field inhomogeneity to the EPI image reconstruction, and then retrospectively obtain distortion-free density images using a regularized least-square method. Since the field map and the spin density are both unknown, our method is a two-step approach (i) we estimate the position of the subject (e.g. a human head) by registering the anatomical dataset to an initially reconstructed EPI slice [1] and generate a field map using the susceptibility ( $\chi$ ) map transformed with the estimated registration parameters. The estimated  $\chi$ -map is generated from a susceptibility voxel convolution (SVC) kernel [2] as described below. (ii) Once the field map is constructed using the  $\chi$  map, we construct the transfer matrix of the data acquisition (forward) model, and estimate the unknown spin density of interest.

## Methods

In discrete form, the EPI data-acquisition model of one slice of spin density  $\rho_z(x, y)$  at  $z$  for  $x = 0, \dots, M-1$  and  $y = 0, \dots, N-1$  can be approximately expressed as

$$S_z(k_x, k_y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \rho_z(x, y) \exp \left\{ j\gamma \Delta B_0(x, y, z) \left( \pm k_x \frac{M\Delta t}{2\pi} + k_y \frac{N^2\Delta t}{2\pi} \right) \right\} \exp \left\{ j2\pi \left( \frac{k_x x}{M} + \frac{k_y y}{N} \right) \right\} + \varepsilon(k_x, k_y)$$

where  $S_z(k_x, k_y)$  is the measured (2-D) k-space samples for  $k_x = 0, \dots, M-1$  and  $k_y = 0, \dots, N-1$ ,  $\Delta B_0(x, y, z)$  is the unknown field map,  $\varepsilon(k_x, k_y)$  is assumed to be an i.i.d. zero-mean complex noise, and  $\Delta t, \gamma$  are the dwell time and the gyromagnetic ratio, respectively. As seen in the data model,  $\Delta B_0$  directly affects the Fourier bases and leads the given measurements to result in non-uniformly sampled k-space data. Taking the standard Fourier transform (with a corrupted set of Fourier bases) of  $S$  makes the reconstructed density image geometrically distorted mostly along y-axis, i.e., phase encoding direction, because  $N^2 \gg M$  as seen in Fig.1(b). Therefore, the reconstruction performance of  $\rho_z$  strongly depends on how accurately we estimate  $\Delta B_0$ . Given that the measured  $\Delta B_0$  of each EPI slice is not available, we generate  $\Delta B_0$  from the  $\chi$ -map of the subject. The computed  $\Delta B_0$  can be obtained by applying a susceptibility voxel convolution (SVC) kernel  $h$  to  $\chi$ , i.e.  $\Delta B_0(x, y, z) = h(x, y, z) * \chi(x, y, z)$ . Assuming that the scanning condition of the MRI machine remains unchanged, i.e., same acquisition session, and the SVC kernel stays constant, with the known subject position, the field map can be computed by transforming the susceptibility map. It is worth noting here that we cannot transform  $\Delta B_0$  directly because the SVC kernel  $h$  is asymmetrical. In our approach, we compute the susceptibility map ( $\hat{\chi}$ ) of the subject from an anatomical (distortion-free) dataset mapped to the segmented brain atlas [1, 3]. After the EPI scan, first we register the anatomical data onto a pilot estimate of  $\rho(x, y, z)$ , in which distortions are present, and obtain the registration parameters.

Using the parameter, we transform the computed susceptibility map ( $\hat{\chi} \rightarrow \tilde{\chi}$ ) and obtain the estimate of  $\Delta B_0$  by convolving an SVC kernel  $h$  to the transformed susceptibility map  $\tilde{\chi}$ :  $\Delta \hat{B}_0(x, y, z) = h(x, y, z) * \tilde{\chi}(x, y, z)$ . Once the field map is available, we estimate  $\rho$  from the data model with the estimated field map. For our convenience, we rewrite the data model in matrix form as  $\underline{s}_z = \mathbf{F}_{\Delta B_0} \underline{\rho}_z + \underline{\varepsilon}$  where  $\underline{s}_z \in \mathbb{R}^{MN \times 1}$  is the measured samples in k-space,  $\mathbf{F}_{\Delta B_0} \in \mathbb{R}^{MN \times MN}$  is the transfer (system) matrix,  $\underline{\rho}_z \in \mathbb{R}^{MN \times 1}$  is the spin density image at  $z$ , and  $\underline{\varepsilon} \in \mathbb{R}^{MN \times 1}$  is the noise image. The underscore represents that the column vectors of the matrix are lexicographically stacked into a

vector (e.g.  $\underline{s}_z = [\dots, S_z(k_x, k_y), \dots]^T \in \mathbb{R}^{MN \times 1}$ ). To find the spin density, a least square estimator with total-variation (TV) regularization,  $C_{TV}(\underline{\rho}_z)$ , was chosen:

$$\hat{\underline{\rho}}_z = \arg \min_{\underline{\rho}_z} \left\| \underline{s}_z - \mathbf{F}_{\Delta B_0} \underline{\rho}_z \right\|_2^2 + \lambda C_{TV}(\underline{\rho}_z)$$

with a regularization parameter  $\lambda$ .

## Simulations

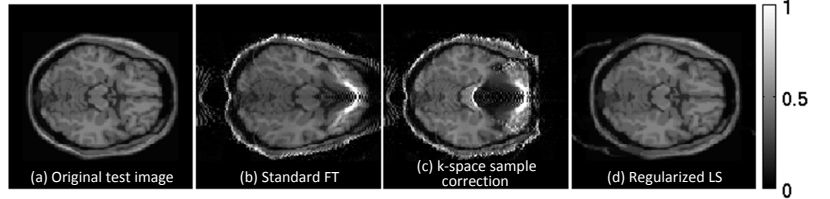
Using a spin density MR image (128 × 128) shown in Fig.1(a) and its computed field map Fig.2, we first generated a k-space samples by using the data model with the dwell time 5 ms. No noise is added in this simulation. Then, we reconstruct the density map by three methods, standard Fourier transform, the k-space sample correction with the field map (also known as the field-map based method), and the regularized least square estimator with TV. The reconstructed images are shown in Figs.1(b)-(d), and Fig.3 shows absolute differences between the original and reconstructed images. The difference images clearly show the advantage of the regularized LS estimator. Note that  $\Delta t$  in the upper limit demonstrates a distortion with which the original structures are severely damaged, and they are beyond restoration by any spatial domain approaches.

## Discussion and Future Works

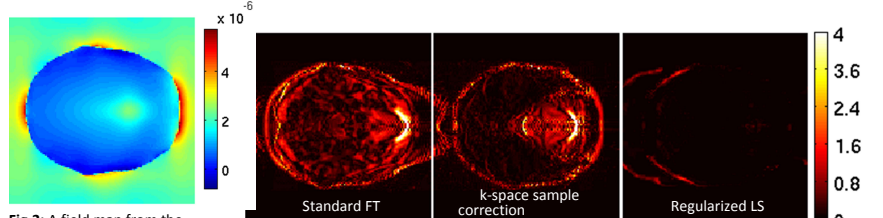
The simulation shows that the approach works effectively with an accurate estimate of the field map. Currently, in our approach, we neglect the other possible distortion factors, such as the coil sensitivity and the MRI system specific error. It is one of our immediate future work to examine how the errors of the field map estimate affect the final reconstruction of EPI, and to take the coil sensitivity and the system specific error into the estimation.

**References** [1] Yoder, DA, et al., *Mag. Res. Img.*, 2004, 22, 315; [2] Pluim, JPW, et al., *IEEE-TMI*, 2003, 22, 986; [3] Cocosco CA, et al, *Neuroimage* 1997, 5, S425

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**Fig.1:** An EPI reconstruction simulation. The images (a) is a spin density MR image, 128×128, and the images (b)-(c) are reconstructed density image from a simulated k-space sample by standard Fourier transform, k-space sample correction, and (d) the regularized least square estimator with TV.



**Fig.2:** A field map from the computed  $\chi$ -map using the segmented brain atlas at 1.5 T.

**Fig.3:** Absolute difference images between the original density image and the reconstructed images shown in Figs.1(b)-(d), respectively.