Nonlinear inverse reconstruction for T2 mapping using the generating function formalism on undersampled Cartesian data

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Introduction

Quantitative evaluations of the T2 relaxation time are of high importance for diagnostic MRI. Standard T2 mapping procedures rely on the time-demanding acquisition of fully-sampled MSE datasets. Recently proposed nonlinear inversion strategies allow for T2 mapping from undersampled data by exploiting a mono-exponential signal model [1,2]. However, in the presence of B1+ inhomogeneities and non-ideal slice profiles, the echo train of true MR data usually strongly deviates from the idealized model [3,4]. The reconstructed T2 maps therefore contain systematic errors, even for fully-sampled data sets. Using the method [1] on undersampled MSE data, the strong model violation of the first echo can provoke artifacts in the reconstruction as well as a systematic deviation in the results for different acceleration factors. Consequently, the first echo has been discarded in [1] which seems common in practice [5]. Recently a new analytical formula has been proposed [6], which models the MSE signal much more accurately than a mono-exponential curve (Fig.1). The approach has been extended for slice selective sequences and its quantitative superiority demonstrated in [7]. This work evaluates the combination of the model in [7] with the nonlinear inversion approach in [1] to allow for accurate T2 reconstructions from highly undersampled Cartesian data.

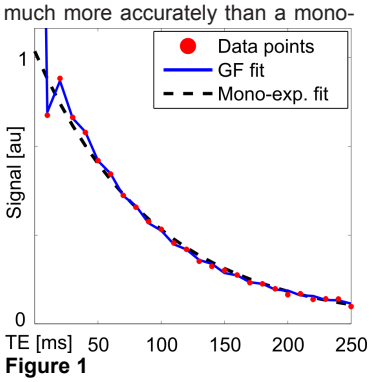

Figure 1

Theory and methods

The so-called generating function formalism (GF) is defined in the z-transform domain:

$$G(z)=\frac{M}{2}\left(1+\sqrt{\frac{(1+zk_2)[1-z(k_1+k_2)\cos\alpha+z^2k_1k_2]}{(-1+zk_2)[-1+z(k_1-k_2)\cos\alpha+z^2k_1k_2]}}\right) \quad (1) \qquad G_{SP}(z)=\frac{1}{Q}\sum_{i=1}^{Q}G(z,\alpha_i) \quad (2).$$

Here, M is the spin-density, k_1 = exp{-τ/T1} and k_2 = exp{-τ/T2} are the relaxation terms, α is the refocusing flip angle, τ the inter-echo spacing and T1 / T2 the relaxation times. z denotes a complex variable in the z-domain. Eqn. 2 accounts for non-ideal slice profiles [7]. Evaluation of this term for z = exp{iφ} (φ = 0…2π) and, thereon, applying a DFT in z-direction yields a discrete time signal corresponding to the echo amplitudes at sampling times nτ. The samples from an undersampled k-space can therefore be modeled by Eqn. 3, where P denotes the sampling pattern, F_z is a one-dimensional DFT in z-direction, F_{xy} the DFT in the two spatial directions and C_c the complex sensitivities of the coil elements c.

$$\hat{s}_{TE,c}(x)=PF_{xy}\{C_cF_z[G_{SP}(x)]\} \quad (3) \qquad \Phi(x)=\frac{1}{2}\sum_{TE}\sum_{c}\left\|\hat{s}_{TE,c}(x)-s_{TE,c}\right\|_2^2 \quad (4) \qquad x=\begin{pmatrix}M\\T2\end{pmatrix} \quad (5)$$

The cost function (4) measures the difference between the modeled samples $\hat{s}$ and the actually acquired samples s. Reconstruction of the parameter maps T2 and M can be performed by minimizing (4) for the unknown vector (5).

We used the CG-Descent algorithm [8] for the minimization of the cost function ϕ. The partial derivatives of ϕ were balanced with respect to M and T2 by additional scaling coefficients. For this proof-of-principle study, the scaling has been initialized with heuristically chosen scalar values. To reduce truncation artifacts, the signal in z-domain was evaluated at 64 points. Only the first 25 points were further used in the time-domain.

In contrast to traditional pixel-wise fitting, regions with implausible signal behavior or long T2 (i.e. CSF) cannot simply be excluded from the reconstruction. We observed that those regions tend to provoke very high values in the gradient of ϕ. To prevent the algorithm from neglecting other regions, we implemented a dynamic validity mask to reduce the gradient scaling at respective regions. As those regions are initially unknown for undersampled data, reconstruction was performed in a 3-step approach with a fixed number of 3 × 200 iterations. After every step the validity mask was updated, dampening the gradient at regions with an intermediate T2>120 ms.

MRI experiments were conducted at 3 T (Siemens Tim Trio) using a 12-element head coil. The field map α was acquired using the method by Bloch-Siegert shift (Gaussian pulse, $B_{1,peak}$=0.11 G, K_{BS}=21.3 rad/G²/ms, 8s duration, f_{OR}=8kHz). T1 values were obtained using a TIR sequence (TR/TE/TI/contrasts 7s / 100-3100 ms / 6). The data samples $s_{TE,c}$ were acquired using a MSE sequence (TR/TE/contrasts 4000 ms / 10 ms / 25). Undersampling was performed with the blocked sampling scheme from [1] selecting respective k-space lines from the fully sampled data. The coil profiles C_c were calculated in a pre-processing step employing the method [9] on a fully sampled composite k-space of AF subsequent echoes.

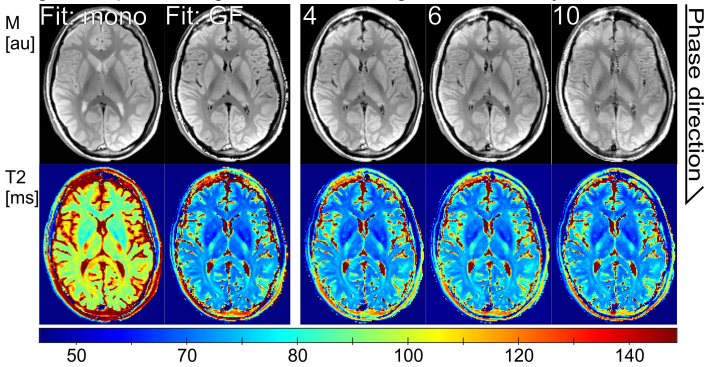

Figure 2

Results

Fig.2 (left) shows spin-density and T2 maps reconstructed by pixel-wise fitting of fully-sampled magnitude images from the sum-of-squares of all channels. The traditional mono-exponential fit is compared to the GF method and highlights the quantitative difference between the two approaches.

Fig.2(right) illustrates reconstructions using the proposed method on undersampled MSE data with different acceleration factors (AF). While the quality of the spin-density map clearly degrades with higher AF, the reconstructed T2 maps are in remarkably good agreement with the fully-sampled reference up to the highest AF of 10.

References [1] Sumpf TJ et al. JMRI 2011;34:420-428, [2] Block KT et al, IEEE Trans Med Imaging 2009;28:1759-1769 [3] Majumdar S et al. MRM 1987;4:203, [4] Crawley AP et al. MRM 1987;4:34, [5] Mosher TJ et al. Osteoarthr. Cartil 2010;18:358, [6] Lukzen NN et al. JMR 2007;185:71, [7] Petrovic A et al. Proc ISMRM 2010, 2749. [8] Hager WW et al. SIAM J Optim. 2006;16:170-192. [9] Uecker M et al. MRM 2008;60:674-682