An improved algorithm for the estimation of multi-component T_2 distributions

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INTRODUCTION Recently there has been increasing interest in estimating the distribution of T_2 relaxation times contributing to a magnetic resonance signal. This distribution has been used to study white matter diseases such as multiple sclerosis [1]. Recovering the distribution for every pixel in an MR image is a challenging problem due to the relatively small number of measurements and low SNR. Previous approaches include fitting a large number of decaying exponentials using the non-negative least-squares (NNLS) algorithm [2]. Alternatively, a parametric model consisting of discrete components was proposed in [3] and subsequently tested in [4]. Although the discrete model is promising, it has not demonstrated reliable performance on clinical images at realistic SNR values.

In this work, we consider the discrete model of multi-component T_2 relaxation and examine the corresponding least-squares cost function. We show that local minima and algorithm initialisation are problematic for a naive optimisation algorithm. We develop a Bayesian algorithm that overcomes these limitations and provides reliable T_2 estimates.

THEORY Under the assumption of discrete T_2 components, the measurements at time t_i are described by Eq. (1), where v_i is assumed to be a Gaussian random variable. A common approach to estimating the component weights, $w_1, \ldots, w_m$, and relaxation rates, $\tau_1, \ldots, \tau_m$, is a gradient-based optimisation algorithm to minimise the sum of squared errors, Eq. (2).

$$y(t_i) = \sum_{j=1}^{m} w_j e^{-t_i/\tau_j} + v_i \qquad (1)$$

$$\underset{w_1,\ldots,w_m,\tau_1,\ldots,\tau_m}{\arg\min} \sum_{i=1}^{n} \left(y(t_i) - \sum_{j=1}^{m} w_j e^{-t_i/\tau_j}\right)^2 \qquad (2)$$

To visualise the cost function in Eq. 2, we considered a two component model with fixed weights and calculated the sum of squared errors for different pairs of the two components (τ_1 and τ_2). Figure 1 displays the resulting cost function for a single realisation of Eq. (1) with an SNR of 100. The cost function has local minima and large flat regions meaning a gradient-based optimisation algorithm initialised away from the true values may not converge to the global solution. The problem is more pronounced for a three-component model and/or decreased SNR.

METHODS We adopt a Bayesian framework that leads to a numerically robust algorithm and allows us to incorporate prior information about the biological tissue. The challenge of approximating the posterior for a relatively wide prior and narrow likelihood is overcome using a technique known as progressive correction [5]. We apply the same principle of 'flattening' the likelihood and iteratively correcting the posterior. Each iteration, the flattening is reduced and the posterior converges to the true one. To maintain tractability, we use a linearised likelihood and compute a Gaussian approximation of the posterior rather than a Monte Carlo approximation as in [5]. The algorithm is represented in Figure 2.

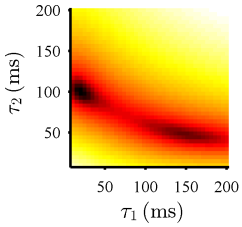

Figure 1: Cost function of a least-squares optimisation algorithm.

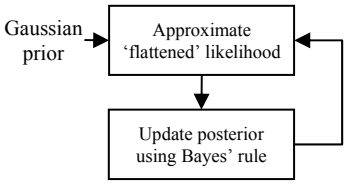

Figure 2: Proposed algorithm; the posterior approximation remains Gaussian each iteration.

The proposed estimation algorithm was tested with simulated and experimental data. All calculations were performed using MATLAB (The Mathworks, Natick, MA). **Simulated Data:** Synthetic data was generated for a single pixel using Equation (1) with a true distribution consisting of two modes: a fast mode ($w_1 = 0.3$, $\tau_1 = 20$ms) and a slow mode ($w_2 = 0.7$, $\tau_2 = 100$ms), typical of a voxel located in white matter. Measurements from 24 echoes were simulated starting at 10ms and spaced 10ms apart. The procedure was repeated to generate 1000 independent realisations for each SNR value. **Experimental Data:** Ex-vivo measurements of a sheep brain suspended in 4% PFA were performed using a 4.7T Bruker BioSpec small bore MRI scanner fitted with a high performance gradient set. A multi-echo CPMG sequence with 24 echoes was used with a 10ms first echo time and echo spacing of 10ms. The FOV=8.96cm × 6.72cm with a matrix size of 256×192. A single mid-axial slice of 2mm was acquired using a volume transmit/receive coil.

T_2 relaxation times were estimated using a gradient-based MATLAB optimisation algorithm (`lsqnonlin`) and the proposed Bayesian algorithm. For simulated data, the procedure was repeated for each noise realisation and the empirical root mean square error (RMSE) was calculated. The optimisation algorithm was initialised with two modes at 20ms and 100ms with equal weights.

RESULTS Figure 3 displays RMSE calculated from the simulation results and illustrates the robust performance of the Bayesian algorithm over a wide range of SNR values. At high SNR, both the Bayesian and gradient-based algorithms successfully find the global minimum. However, for moderate to low (though clinically realistic) SNR, the gradient-based algorithm fails to converge to the correct solution and exhibits a sharp degradation in performance. Figure 4 presents the T_2 estimates from the experimental data. Figures 4a and 4b display the 'fast' mode of the two-component model for the gradient-based and Bayesian algorithm, respectively. Likewise, Figures 4c and 4d display the 'slow' modes for the two algorithms. Although a large number of pixels contain similar T_2 estimates for both algorithms, a closer inspection of the gradient-based results, in Figure 5, reveal many pixels with very large or small T_2 components, which are unlikely to represent the true quantities. Conversely, the proposed algorithm provides consistent estimates within the expected range.

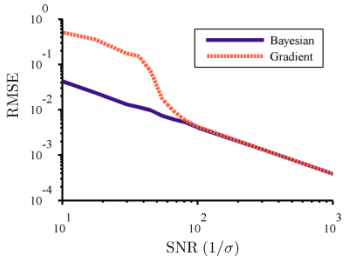

Figure 3: The RMSE of a gradient-based algorithm and the proposed algorithm.

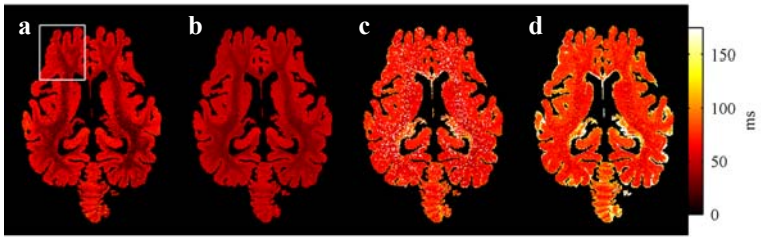

Figure 4: Estimated T_2 maps of the slow mode for (**a**) the gradient-based algorithm and (**b**) the proposed Bayesian algorithm. (**c**) and (**d**) are corresponding maps of the fast mode.

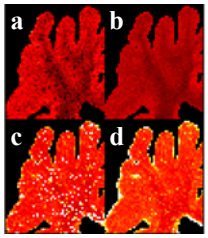

Figure 5: Magnified maps corresponding to Figure 4 for the region marked in 4a

CONCLUSION We have proposed a new algorithm that provides accurate estimates of the weights and T_2 relaxation times of a discrete component model. The algorithm overcomes the limitations of traditional gradient-based approaches associated with low SNR and algorithm initialisation. The reliability of the algorithm on clinically realistic data increases the utility of a discrete component model to assess brain tissue structure.

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