# SNR Requirements for T1 and T2 Estimation using bSSFP

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**Introduction** Recent parameter estimation techniques based on the Freeman-Hill formula [1] for balanced steady-state free precession (bSSFP) have been proposed to remove banding artifacts and estimate off-resonance maps from multiple phase-cycled bSSFP images [2-3]. An intriguing extension of these algorithms is the simultaneous estimation of $T_1$, $T_2$, resonant frequency, and frequency-independent (i.e., banding-free) signal from a set of phase-cycled bSSFP images. Due to the high SNR efficiency of bSSFP, this approach could be very beneficial for rapid parameter estimation, provided that accurate estimation is possible using a moderate number of phase-cycled images. In this work, we present the negative result that the conditioning of the signal equation is such that accurate estimation of $T_1$ and $T_2$ using a moderate number of phase-cycled bSSFP images requires prohibitively high SNR. We derive theoretical limits for the minimum variance of $T_1$ and $T_2$ estimates using the Cramér-Rao Bound (CRB), and, using these limits, we present the SNR requirements to achieve 5% precision using four phase-cycled bSSFP images.

**Theory** The Freeman-Hill formula is given in Eq. 1, where α is the flip angle, $\theta = 2\pi \mathrm{TR} f$, with $f$ the resonant frequency, $\theta_n = \theta + \Delta\theta_n$, with $\Delta\theta_n$ the radio-frequency phase increment, $M_0$ is the equilibrium magnetization, and K is the coil sensitivity. The CRB gives an expression for the minimum variance of an unbiased parameter estimate. For parameters $T_1$, $T_2$, θ, and $KM_0$, the CRB was derived using the Slepian-Bangs formula [4]. The SNRs required to obtain 5% precision of the estimates of $T_1$ and $T_2$, respectively, were then calculated, with precision being defined as the standard deviation relative to the true parameter value. SNR was defined as

$$SNR = 1/\sigma \sqrt{1/N \sum_n |S_n|^2},$$

where $\sigma^2$ is the noise variance of each bSSFP acquisition.

**Methods** Numerical simulations were performed using Matlab and the following parameters: TE = 5ms, TR = 10ms, α = 30°, θ = 0°, $KM_0 = 1$, $\Delta\theta_n = [0, \pi/2, \pi, 3\pi/2]$, $T_1$ = 100-3000ms (1000 values), $T_2$ = 5-200ms (200 values).

**Results** Figures 1 and 2 show the theoretical bound for the minimum SNR required to obtain a standard deviation of the estimates of $T_1$ and $T_2$ that is at most 5% of its true value. The bounds are computed using the CRB and therefore hold for any unbiased estimator. To estimate the relaxation parameters of white matter and gray matter in the brain at 1.5 T, we see that an SNR of roughly 100 (40 dB) is needed. This SNR is above what is typically obtained in a bSSFP scan. While Figs. 1-2 assumed a constant θ, Fig. 3 plots the CRB of T1, T2, θ, and $KM_0$ over a range of θ values from 0 to 360°, using an average SNR of 32 (30 dB). The true parameter values were selected to represent white matter at 1.5 T: $T_1$ = 675ms, $T_2$ = 75ms, and $KM_0 = 1$. The precisions of θ and $KM_0$ are approximately 5% of the true parameter values, while the precisions of $T_1$ and $T_2$ are approximately 13% and 12% of the true parameter values, respectively.

**Discussion and Conclusion** The CRB analysis has shown that, from a theoretical standpoint, $T_1$ and $T_2$ are much more difficult to estimate than θ or $KM_0$. In practice, additional factors such as magnetization transfer effects and motion will contribute additional complications for in vivo studies. However, even neglecting these factors, the SNR required to precisely estimate $T_1$ and $T_2$ from a set of four phase-cycled bSSFP images is higher than what is typically obtained in vivo. The SNR requirements for 5% precision can be reduced by acquiring a larger number of phase-cycled bSSFP images, but this comes at the expense of longer scan times. For example, using 32 phase-cycled bSSFP images reduces the SNR requirement for white matter $T_1$ and $T_2$ estimation to 29 dB ($|S_n|/\sigma$ =28). We therefore conclude that, in general, phase-cycled bSSFP is not a reliable method to estimate $T_1$ and $T_2$ in vivo on a clinical system.

**References**

[1] Freeman, *et al.* JMR 4:366-383, 1971. [2] Björk, *et al.* Proc 19th EUSIPCO 1000-1004, 2011.
[3] Santini, *et al.* Proc. 18th ISMRM 3089, 2010. [4] Stoica, *et al.* Spectral Analysis of Signals, 2005.

$$S_n = KM_0 c e^{i\theta \mathrm{TE/TR}} \frac{1 - a e^{-i\theta_n}}{1 - b\cos(\theta_n)}$$
$$a = E_2$$
$$b = E_2 \frac{1 - E_1\cos\alpha - E_1 + \cos\alpha}{1 - E_1\cos\alpha - (E_1 - \cos\alpha)E_2^2}$$
$$c = ie^{-\mathrm{TE}/T_2} \frac{(1 - E_1)\sin\alpha}{1 - E_1\cos\alpha - (E_1 - \cos\alpha)E_2^2}$$
$$E_1 = e^{-\mathrm{TR}/T_1}$$
$$E_2 = e^{-\mathrm{TR}/T_2}$$

**Equation 1**. Freeman-Hill Formula

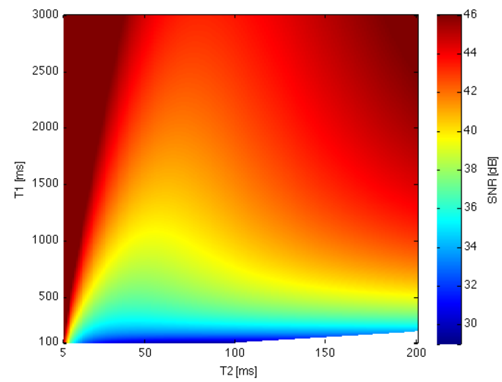


**Figure 1**. SNR required to obtain 5% precision on $T_1$ estimation using 4 phase-cycled SSFP images.

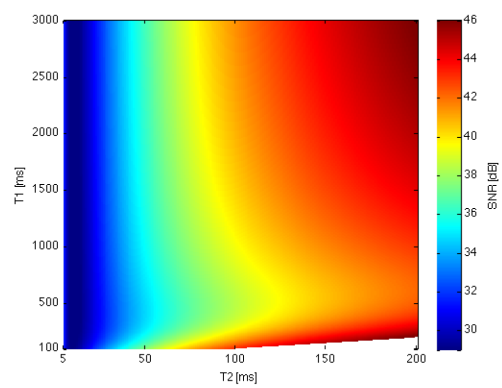


**Figure 2**. SNR required to obtain 5% precision on $T_2$ estimation using 4 phase-cycled SSFP images.

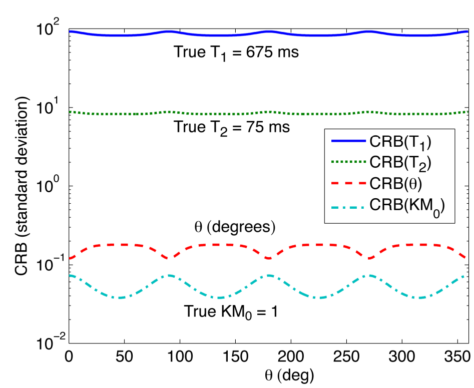


**Figure 3**. CRB of $T_1$, $T_2$, θ, and $KM_0$ for an average SNR of 32 (30 dB).