Reliable fitting of phase data without unwrapping by wrapping the fit model

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Introduction

Analyzing phase data is important for many applications in MR research. In particular, unwrapping 2D or 3D phase data is becoming increasingly important (e.g., SWI, QSM). Nevertheless even the simpler case of 1D phase unwrapping still plays an important role for many pre-processing steps in image reconstruction, e.g., EPI ghost correction [1] or calculating field maps from multi-echo data[2]. In these latter cases a model is often fitted to the 1D phase data to extract fit parameters (e.g., slope, offset) from the spatial or temporal phase course. Successful unwrapping of 2π phase jumps, however, is crucial for such an analysis. In the present work we aimed to develop a robust method for fit parameter extraction which uses a reverse approach to fit a known model to a 1D phase distribution by wrapping the fit model function while avoiding unwrapping the data.

Methods

Phase unwrapping aims to detect and correct phase jumps between $-\pi$ and π in order to create a continuous phase distribution $\phi(x)$ so that a model *m(x)* can be fitted to the phase data. If the model *m(x)*, however, is known *a priori*, unwrapping can be avoided by wrapping *m(x)* instead.

The wrapped, bounded fit is obtained by minimizing a cost function *K* which calculates the deviation of the fit *m(x)* from the measured phase data $\hat{\varphi}(x)$ for a set of *N* fit parameters $f_1 \ldots f_N$

$$K(f_1,\ldots,f_N) = \sum_x \left| \mathrm{wrap}[m(x,f_1,\ldots,f_N)] - \hat{\varphi}(x) \right|.$$

The `wrap` operator wraps *m(x)* onto the interval of $[-\pi,\pi]$, thus matching the wraps of the input phase $\hat{\varphi}(x)$. To obtain start values for the minimization of the cost function an *N*-dimensional cost matrix is calculated for a reasonable range of discrete values of the fit parameters $f_1 \ldots f_N$. The minimum of this cost matrix is then provided as start parameters for the minimization of *K*.

For a simple linear model

$$m(x) = f_1 \cdot x + f_2$$

the cost function becomes

$$K(f_1,f_2) = \sum_x \left| \mathrm{wrap}[f_1 \cdot x + f_2] - \hat{\varphi}(x) \right|,$$

which can be easily and quickly minimized for the parameters f_1 and f_2 (slope and offset respectively).

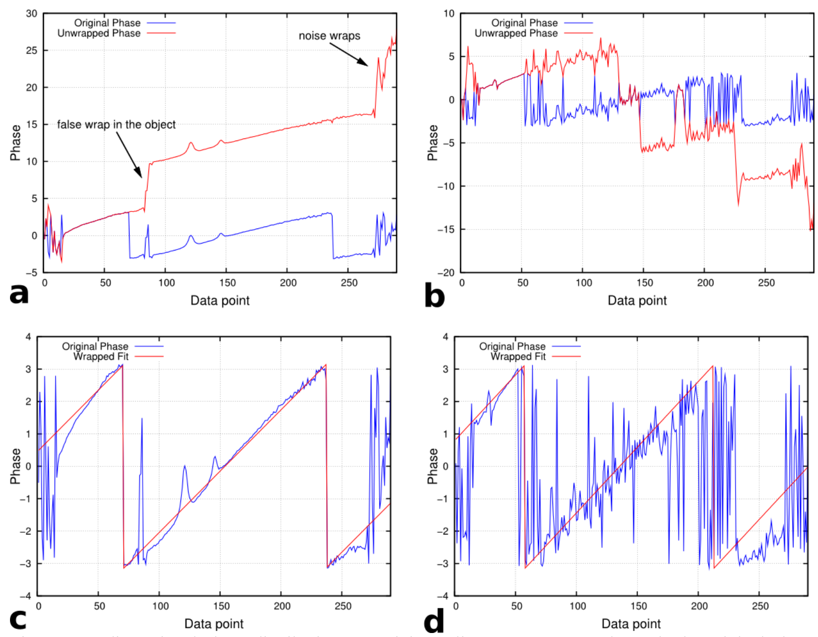

Fig. 1. One dimensional phase distributions containing a linear component. Shown is the original phase without and with conventional unwrapping in a) and b), whereat the quality of the phase in b) is severely degraded by noise. In both cases phase unwrapping fails due to object inhomogeneities or noise, making a fit of the linear component impossible. Shown in c) and d) are the fit results of our proposed method for the phase data of a) and b) respectively, demonstrating that the wrapped fit is able to find a linear component. Even with the very noisy data in d) a linear fit was possible.

A typical calibration scan for EPI ghost correction was used to obtain demonstration data. With this scan, an EPI trajectory is played out without phase encoding blips, effectively acquiring the central k-space line multiple times. After FFT of the single readouts, data from even and odd echoes was combined to obtain the phase difference between the readout directions. Since the even and odd echo centers are shifted in opposite directions the resulting phase difference shows a linear dependency. Phantom data was acquired on a 3T Siemens Trio system using the following acquisition parameters: 200 mm^2 FOV, 200 kHz acquisition bandwidth, 400 data points for each readout and 1.8 ms echo time for a single readout. As comparison 1D phase unwrapping with a jump tolerance of π was performed using in-built functions of the Matlab software package.

Results

Figure 1 shows how the wrapped fit method performs for two exemplary phase distributions (a and b respectively). Clear visible is a linear component which however couldn't be retrieved by conventional unwrapping because the 1D phase unwrapping failed due to phase inhomogeneities within the object or noise. With the proposed method a linear fit could be applied in both cases since the wrapped fit method is insensitive to faulty data points and data degraded by noise. Even if there was substantial noise present in the data the wrapped fit performed very well. In both cases the normal approach of fitting the unwrapped phase would have failed.

Discussion

The presented method has shown to be a reliable alternative for fitting phase data. Even much degraded data can be properly fitted since no phase unwrapping is necessary. Although we have only shown a linear fit model as example it is conceivable that the same approach also works for non linear models. Application of our method is especially recommendable when a high accuracy with few outliers is desired, for example when phase data is used to obtain calibration data which is later applied to an entire dataset. When applied to fully automated algorithms which do not require any user input the reliability gained by the proposed method is highly advantageous. This is not only due to increased robustness towards even very strong noise, but also because our method is insensitive to single outliers or faulty data points.

References

[1] Bruder H, et.al., MRM 1992, [2] Schmithorst VJ, et.al. IEEE Trans Med imag, 2001