

Iterative nonlinear encoding magnetic gradient phasor optimization for single readout parallel imaging

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Introduction: Recently, nonlinear encoding gradient methods have been explored using the PatLoc¹ or the O-space approach². In particular, the O-space approach uses a reconstruction with an over-complete basis formed during repetitions (TRs) at different center placements. Compared to linear gradients, a search through the higher order gradient sets allows more freedom in the choice of reconstruction bases³. In the following study, an optimal over-complete spatial basis is determined through an iterative search of the achievable encoding bases (phasors) and minimum dB/dt transitions are selected for single readout parallel imaging.

Method: The MRI signal equation for algebraic reconstruction of an object ρ using receiver coil sensitivities C_l with multiple nonlinear gradients G_m , where each G_m represents a different magnetic encoding shape, $S_{l,t} = \int \rho(\vec{r}) C_l(\vec{r}) e^{-i2\pi G_{m,n}(\vec{r})t} d\vec{r}$, where $l = 1 \dots n_c$, the number of coils, and $t = 1 \dots N_s$, the number of samples. The signal equation may be represented in matrix form, $\mathbf{s} = \mathbf{B} \rho$, where \mathbf{s} is a vector containing the measurements and ρ is the vectorized image. Each row of \mathbf{B} is represented by a phasor equal to the product of a coil sensitivity profile and phase evolution imposed by the gradient at a given time, i.e. the phasor $\phi_{l,t,m} = C_l(\vec{r}) \cdot e^{-i2\pi G_m(\vec{r})t}$.

Traditionally, MRI uses three orthogonal linear gradients ($m=3$) to spatially encode an object. In this study, the linear terms plus two nonlinear gradients are considered ($m=5$). The goal in phasor optimization is to efficiently encode the object while preserving smoothness between phasors for minimal dB/dt during readout.

A variation on the matching pursuit algorithm⁴ determined phasors based on orthogonality to previously selected phasors and smoothness in the form of minimum absolute difference. To simplify the search landscape, a library of phasors was built from a product of the available receiver coil sensitivities and achievable phase winding generated by a planned second order gradient system. The gradient set includes the first and second order non-degenerate in-plane spherical harmonics, by common name: X, Y, XY, C2, and Z2, an optimal set⁵. Receiver coil sensitivities were simulated based on a microstrip array coil⁶ and reconstructions performed via the Kaczmarz algebraic reconstruction technique⁷. Whole body noise was injected at 5%.

Results: Images were reconstructed at a fixed bandwidth for varying sampling durations from 25ms to 50ms. The sum of squared errors decreases as the number of encoding phasors is increased. Furthermore, a decrease in over 50% of the maximum slew rate is observed for the iterative method versus a conventional echo planar imaging readout (87 T/m/s versus 180 T/m/s).

Discussion: The iterative method presented here suggests an efficient method of spatial encoding that allows smooth gradient trajectory single readout images. While there are many possible gradient trajectories for efficient encoding, a systematic evaluation of basis orthogonality and smoothness at each time point may yield better results than non-optimized methods.

Regarding noise, there remains a trade-off between sampling and noise amplification, which may be another dimension of optimization.

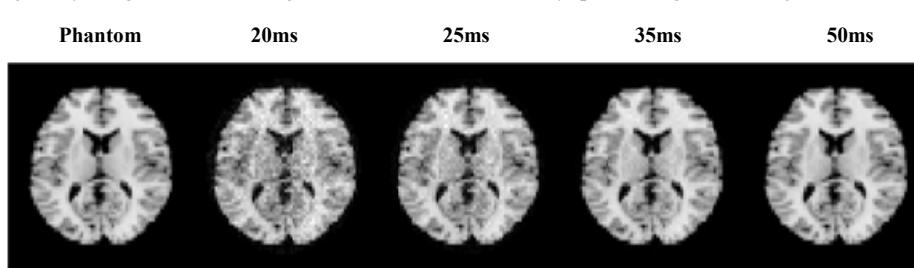


Fig 2. Picture from left to right, reference phantom and reconstruction with increasing number of samples (total acquisition time is listed above each image) in this single shot acquisition. The sampling bandwidth is fixed at 256 kHz while the sampling duration is increased.

References: ¹Stockmann et. al. Magn Reson Med. 2010; 64: p. 447-456. ²Hennig, et. al. ISMRM 2007, 453 ³Lin, F-H, et al., Proc. Int'l. Soc. Magn. Reson. Med., 2011, p.480. ⁴Mallat, S.G. IEEE 1993; 41:12. ⁵Tam, L.K., et al., Proc. Int'l. Soc. Magn. Reson. Med., 2011, p.721. ⁶Lee, RF, et. al. Magn. Reson. Med 2004; 51:172. ⁷Herman G.T. et. al. J. Theor. Biol. 42:1. **Acknowledgements:** This work supported by NIH BRP R01 EB012289-01 and a National Science Foundation Graduate Research Fellowship.

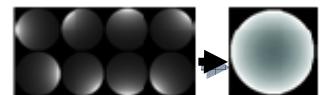


Figure 1a. Eight element microstrip array coil

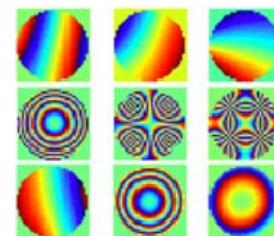
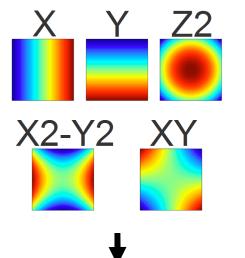


Figure 1b. An iterative search through the possible phasors generated by a nonlinear gradient set (top) yields an efficient and smooth encoding basis. (bottom) A sample of the encoding basis phasors from the start, middle, and end of the readout (first, second, and third row, respectively)