Autocalibrating Reconstruction for Non-bijective Encoding Fields

Ethan M. Johnson[1], John M. Pauly
[1]*Electrical Engineering, Stanford University, Stanford, CA, United States*

Introduction The concept of using spatially-nonlinear, non-bijective magnetic fields for MR image encoding to taylor resolution and SNR has inspired development of novel reconstruction techniques [1]-[4]. However, these techniques make explicit use of receive coil sensitivity for reconstruction, by a SENSE-like algorithm [1] or by approximate solution of the 'encoding equation' ($\mathbf{s} = E\mathbf{m}$) [2]. Since accurate receive sensitivity maps can be challenging or onerous to obtain, an auto-calibrating reconstruction technique for non-bijective fields that obviates need for explicit maps would be valuable in making such imaging methods practical. Here, focus is given to encoding by a 'gradient array' [5] [6] applied in y and a conventional (linear, bijective over FOV) field applied in x (fig. 1). Consideration is given afterward to other configurations.

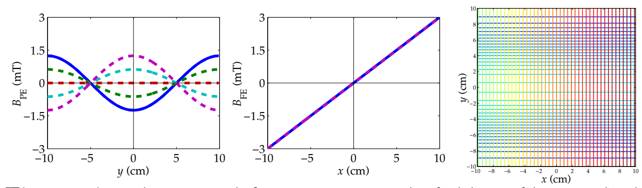


Figure 1: *Phase- and frequency-encode field profiles applied for 5 spin-warp PEs and readouts (same B_{FE}). PE field is from a gradient array* [5] *with 1 cycle over 20cm FOV. Resolution 'map' shows pixel sizes as enclosed areas, and alike-colour-enclosed regions map to the same encoded position (i.e., alias!).*

Theory For simplicity, the fields are applied for 'Cartesian' acquisition, i.e., modulated in amplitude by a spin-warp pulse sequence (fig. 1). This samples the 'encoding k-space' on a Cartesian grid, which is the k-space of an object warped by the field non-linearity and aliased by the field non-bijectivity. Regions with similar polarisation of the encoding field (same sign for dB/dy) alias together in the same manner as in an undersampled 'conventional' image. Regions with opposite polarity also alias but are reversed in spatial orientation. Spatial-reversal is equivalent to frequency-reversal, so the k-space for these regions is reversed. If data acquired by this protocol are upsampled to match a full image FOV, then they relate to the k-space of a spatially-warped image that has been R-fold undersampled and has opposite-polarity-encoding-field regions flipped, where R the number of like-encoded points. (E.g., fig. 1: R=2.)

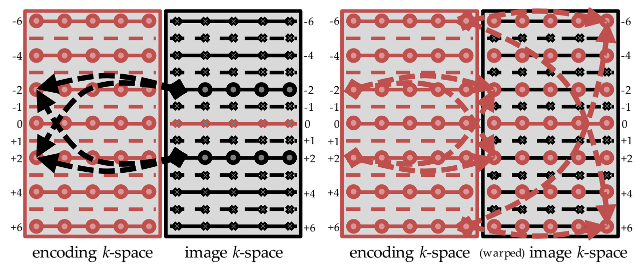
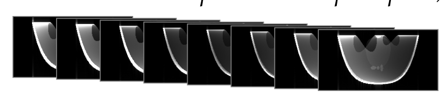


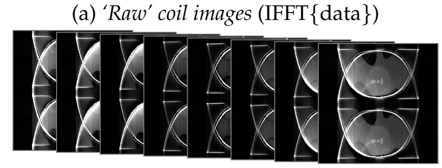
Figure 2: *Weights calibrated by regressing the ± 2 lines in enc. k-space against 2 ACS from img. k-space (left). Undersampled k-space for warped img. is synthesized in pairs from enc. k-space using weights (right). Only 2 lines from img. k-space, acquired with a linear bijective field in y, are used to calibrate. (1 coil data set—not realistic! Enc. k-space shown upsampled.)*

Methods The non-bijective field is used to phase-encode, matching the reduction in size of 'locally-bijective' fields-of-view to the reduction in encoded acquisitions. The image distortions that result can then be undone in three stages: (1) for each coil re-orient the reversed frequencies by re-combining coil data using autocalibrated weights (fig. 2), (2) combine frequency-unfolded data with calibration data and fill in missing k-space locations (GRAPPA [7]) and (3) resample the image to a uniform spatial grid. The weights 'kernel' size for frequency-unfolding is a free parameter. To differentiate aliased regions for frequency re-orientation, the auto-calibration signals (ACS) must be acquired by a bijective field. If some loss of reconstruction fidelity is tolerable, this can be a conventional linear gradient, even though the non-bijective field is not piecewise linear! The spatial warping is corrected by spatial re-sampling according to the deviation from linearity of the field, and intensities corrected by the spatial derivative of the field.

Acquisition simulation and image reconstruction are performed using Matlab. Data are generated by specifying a pulse sequence and pair of encoding fields, accruing phase at each voxel of a numerical (Shepp-Logan) phantom according to its position r and the time t as $\int_0^t \gamma B(r,\tau) d\tau$, and the aggregate signals for 8 receive coils are sampled during the conventional spin-warp readout time. Reconstruction is performed by the procedure above, and the final step is a sum-of-squares combination.



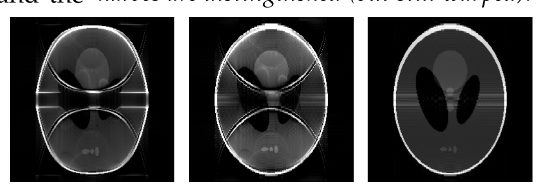
(a) *'Raw' coil images* (IFFT{data})



(b) *Frequency-unfolded coil images*

Figure 3: *'Raw' coil images show aliasing and warping: the top half flips and aliases with the bottom, and both halves warp. Unfolded images show correct orientation: the halves are distinguished (but still warped).*

Results Autocalibrated frequency-unfolding re-orients regions of opposite polarity (fig. 3). However, some errors are evident in the final result (fig. 4b), due to calibration of weights for nonlinear-encoded data using linear-encoded ACS—the GRAPPA kernel is 'mis-calibrated'. The reconstruction appears to have 'one big pixel' at its vertical centre; in fact it does, and this is a consequence of the non-uniform resolution induced by the non-linear encoding field (fig. 1).



(a) *k-space interp.* (b) *Resampled* (c) *SENSE (ref.)*

Figure 4: *A full-FOV image is formed by GRAPPA k-space interpolation from the same and additional ACS. Spatial resampling to a uniform grid restores shape. (SENSE reconstruction shown for reference.)*

Discussion This reconstruction procedure makes no explicit use of receive coil sensitivity maps. It requires a bijective encoding field to acquire ACS, but any usable sensitivity map must be measured with bijective fields, so this method is deemed comparable to precedents. Additionally, no explicit delineation of local field polarity is made, nor is image space partitioned into locally-bijective encoding regions, so this technique is extensible to other encoding fields. The pair-wise frequency-unfolding structure is not field pattern-dependent, and the degree of undersampling R is simply the number of 'locally-bijective' fields-of-view, analagous to the acceleration of 'conventional' parallel imaging.

References [1] Schultz et. al, MRM 64:1390-404, 2010. [2] Stockmann et. al, MRM 64:447-56, 2010. [3] Stockmann et. al, ISMRM '11:744, 2011. [4] Schultz et. al, ISMRM '11:481, 2011. [5] Parker et. al, MRM 56:1251-60, 2006. [6] Oppelt, US Pat. 6255821 B1, 2001. [7] Griswold et. al, MRM 47:1202-1210, 2002.