Ensemble Average Propagator Reconstruction via Compressed Sensing: Discrete or Continuous Bases ?

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INTRODUCTION: Sparsity is one of the key ingredient in Compressed Sensing (CS) recovery. The sparsity expresses the idea that a signal contains a small number of non-zero coefficients. Many transforms are known to make a signal sparse as the Discrete Wavelet Transform or the Discrete Cosine Transform respectively used in JPEG2000 and JPEG standards [2]. In Diffusion MRI (dMRI), few studies have been proposed to characterize the sparsity of the Ensemble Average Propagator (EAP) which captures the water diffusion phenomenon. In [1], the authors study the sparsity of five discrete EAP representations but their measures of sparsity depend on the reconstruction method used to estimate the EAP. [5] solves the CS-dMRI problem by applying a gradient as sparse transform. In [6], the authors take advantage of the natural EAP sparsity. In this work, we propose a fair comparison of two classes of representations : The discrete representations, via the Haar, Daubechie-Cohen-Fauveau (DCF) 5-3, DCF 9-7 wavelets bases [1, 2], and the continuous representations, via Spherical Polar Fourier (SPF) [3] and 3D-SHORE [4] bases. Fast discrete transforms have been proposed to model signals in wavelet bases. They have the advantage to give an inverse transform without suffering from information loss. When dealing with SPF and SHORE bases, there is a infinite number of atoms. Hence, we need to truncate them to a given order. Signal modeling is then done via least squares estimation (LSE) with or without regularization. However, continuous representations lead to analytical formulas to estimate other diffusion features [3]. In this work, we study the advantages and disadvantages of these discrete and continuous EAP representations for the first time. Here, the measure of sparsity is not biased by the reconstruction method as in [1].

METHOD: Under the narrow pulse approximation, the EAP in real space, denoted P, and the diffusion signal in q-space, denoted E, are related by a Fourier Transform. In the CS-dMRI problem, we acquire E and try to estimate P. Among others conditions, we want P to have a sparse representation. We call *c* the transform coefficients representing P in a sparse basis. We evaluate the EAP sparsity in fives bases : Three wavelet bases built from 1) Haar wavelet, 2) DCF 5-3 wavelet, 3) DCF 7-9 wavelet, and two other orthonormal bases i.e. the 4) SPF basis and 5) SHORE basis. We generate the diffusion signal from a multi-Gaussian model through six scenarios : One fiber, two 60/70/80/90°-crossing fibers and three 90°-crossing fibers. The signal is built on a cube of size 16*16*16 with a maximum b-value of 6900 s/mm^2. When dealing with bases 1-2-3, we apply a fast Fourier transform to the diffusion signal and then estimate c, via the corresponding discrete wavelet transform. When no fast transform is available (case of bases 4-5), we obtain the diffusion signal coefficients via ordinary LSE. Then, we can get the propagator via an analytical formula from the c coefficients [4]. If we want to do a fair comparison, we cannot use regularization for this problem. However, by considering an overdetermined system (The number of measurements is larger than the number of atoms) we don't need a regularization term. In order to evaluate the sparsity of our bases, we look at the percentage of coefficients necessary to obtain 99% of the absolute cumulative value of all the coefficients. It gives an insight of how many coefficients is needed to correctly reconstruct the EAP. Moreover we compute the normalized mean square error (nmse) between the ground truth EAP P and the estimated EAP Pe in r-space. The nmse is given by $\sum_{i=1..N} \|P(r_i)-Pe(r_i)\| / \sum_{i=1..N} \|P(r_i)\|$. Table 1 shows these results.

RESULTS:

	1 fiber		2 fibers 90		2 fibers 80		2 fibers 70		2 fibers 60		3 fibers 90	
	% coeffs	nmse	% coeffs	nmse	% coeffs	nmse	% coeffs	nmse	% coeffs	nmse	% coeffs	nmse
Haar	54.57	32.83e-6	60.52	36.60e-6	59.77	35.84e-6	60.42	35.26e-6	60.74	34.56e-6	63.96	38.38e-6
5-3 CDF	23.58	16.98e-6	22.41	11.62e-6	26.54	13.01e-6	28.56	14.33e-6	30.27	15.36e-6	22.53	11.02e-6
9-7 CDF	13.45	6.23e-6	14.53	4.60e-6	18.16	5.21e-6	19.85	5.82e-6	20.19	6.47e-6	14.16	3.37e-6
SPF	19.13	130.34e-3	11.74	85.84e-3	35.61	121.62e-3	35.80	150.18e-3	34.09	188.59e-3	11.55	118.29e-3
SHORE	8.72	31.08e-3	5.08	16.19e-3	15.98	18.98e-3	19.85	21.73e-3	26.88	22.62e-3	3.63	14.05e-3

Table 1: Percentage of coefficients necessary to obtain 99% of the absolute cumulative value of all the coefficients and nmse.

In table 1, the dark gray cells represent the results associated with the discrete representations (Wavelet bases) and in light gray cells the results associated with the continuous representations (SPF and SHORE bases). For each scenario, the blue value represents the lowest percentage of coefficients necessary to correctly represent the propagator according to the criteria defined above, and the red value represents the smallest nmse between the ground truth propagator and the estimated propagator. Firstly, we see that the 9-7 CDF wavelet basis and the SHORE basis have the best sparse properties within their respective classes. The reason for the CDF 9-7 efficiency can be inferred from the visualization of the mother wavelets in figures 1, 2 and 3. Indeed, Haar wavelet is a simple step function that obviously cannot well represent other signal than piece-wise signal. CDF 5-3 has a richer structure, which likely catch noise instead of significant information. Of course, other adequate mother wavelets exist and this topic has to be investigated. In the following, we choose SHORE and CDF 9-7 wavelet bases as representatives of their respective classes. The SHORE basis obviously gives the best sparsity rate in case of 1 fiber and several fibers crossing at a medium or large degree (90 to 70 degrees). This is due to the Gaussian-behavior of this basis that adequately models the diffusion process. For 60 degree crossing fibers and lower degrees, the 9-7 CDF wavelet basis better fit the EAP. Why? The low scale wavelets, contributing in the modeling, enable the representation of small details and, thus, low degree crossings. Furthermore, the 9-7 CDF basis gives, by far, the lowest representation error overall (see nmse in red). This proves that fast transform is an important advantage for a basis. Because of the truncation of SHORE basis (as well as SPF basis), we will never reach a nmse as low as the one obtained with the 9-7 CDF wavelet basis.

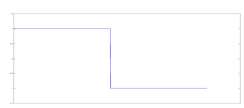

Figure 1 : Haar wavelet

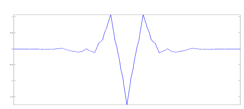

Figure 2 : CDF 5-3 wavelet

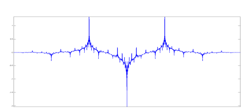

Figure 3 : CDF 9-7 wavelet

CONCLUSIONS: Considering continuous representations, no sparse transform exists. However, due to the Gaussian prior of these bases, they better fit the signal and give lowest sparsity rate than wavelet bases, at least for medium to large degrees of crossing fibers. Another advantage of these bases is the analytical formula used to obtain the EAP, the orientation distribution function (ODF). One drawback is the infinite number of atoms, which forces the bases truncation at a fixed angular an radial order. This truncation adds a modeling error in addition to the reconstruction error. Thanks to its fast transform, the 9-7 CDF basis gives quasi-exact result by modeling the signal with few coefficients. Moreover, 9-7 CDF wavelet basis can handle with smaller details than SHORE basis does. One future work would be the search of the mother wavelet that best fits the diffusion propagator to enhance the sparsity of the basis. To conclude this abstract, we can only suggest a basis depending on the application. If accurate EAP estimation within a cube, as DSI does, is required, then choose the 9-7 CDF wavelet basis. However, when diffusion features are looked for, a continuous representation as SHORE basis is suitable.

REFERENCES: [1] Saint-Amant et al, ISMRM 2011. [2] Mallat, 2009. [3] Assemlal et al, MedIA 2009. [4] Özarslan et al, ISMRM 2009. [5] Menzel et al, MRM 2011. [6] Merlet et al, MICCAI (workshop) 2010