GENERALIZED ABSINTHE WITH SPARSITY-ENFORCING REGULARIZATION

Eric Y. Pierre[1], Nicole Seiberlich[1], Stephen Yutzy[2], Vikas Gulani[3], and Mark A. Griswold[1,3]
[1]*Dept. of Biomedical Engineering, Case Western Reserve University, Cleveland, OH, United States,* [2]*Dept. of Radiology, University of Pittsburgh, PA, United States,* [3]*Dept. of Radiology, University Hospitals of Cleveland, OH, United States*

Introduction: It has been shown that parallel imaging reconstructions are more accurate when working with pre-sparsified undersampled data because there are fewer pixels to unfold from one another (1). In previous works (1,2), we proposed a method to improve GRAPPA results using an image database to identify and remove common image features, such that GRAPPA was only used to reconstruct the unrecognized part of the signal. However, the expected improvements were hampered by the fact that the L2 norm approximation used to recover the known signal minimizes the energy of the unrecognized target image, but not its number of non-zero pixels. In this work, a generalized ABSINTHE framework which enforces sparsity on the unknown signal and is applicable to any accelerated acquisition technique is proposed. We show results for an implementation of the general ABSINTHE framework with SENSE.

Theory: The generalized ABSINTHE framework is described in fig.1. In step1, the entries of the fully-sampled, multicoil image dictionary **D** are combined to yield the least square approximation of the object based on undersampled data **Y**

$$\mathbf{A} = \left(\Phi(\mathbf{D})^H \Phi(\mathbf{D})\right)^{-1} \Phi(\mathbf{D})^H \mathbf{YD} \qquad [1]$$

where **A** is the fully-sampled, multicoil prior image, Φ is the aliasing operator and the H superscript is the Hermitian transpose. Coil images from **A** are combined with the adaptive method (3) which yields the a priori data reconstruction $\boldsymbol{\rho}_a$. The SENSE reconstruction can be calculated iteratively, with regularization to introduce sparsity on unrecognized features:

$$\boldsymbol{\rho}_r{}^i = \arg\min_{\boldsymbol{\rho}} \left\{ \left\| \mathbf{Y} - \Phi(\boldsymbol{\rho}) \right\|^2 + \lambda^2 \left\| \frac{\boldsymbol{\rho} - \boldsymbol{\rho}_a}{\boldsymbol{\rho}_r{}^{i-1} - \boldsymbol{\rho}_a} \right\|^2 \right\} \qquad [2]$$

where $\boldsymbol{\rho}_r{}^i$ is the reconstructed image for the i[th] iteration, $\boldsymbol{\rho}_r{}^{i-1}$ is the previous reconstruction providing an estimate of the target image and λ a fuzzy threshold. The first term in the cost function drives the data consistency, while the second term is the regularization term. Pixel values for the reconstructed unrecognized object $\boldsymbol{\rho}-\boldsymbol{\rho}_a$ are made close to 0 (sparsity) when the estimation of the unrecognized object $\boldsymbol{\rho}_r{}^{i-1}-\boldsymbol{\rho}_a \ll \lambda$, with λ chosen by L-curve method (4). $\boldsymbol{\rho}_r{}^0$ (initialization) is a standard SENSE reconstruction in this implementation. Equation [2] has an analytical solution for SENSE reconstruction well described in (5).

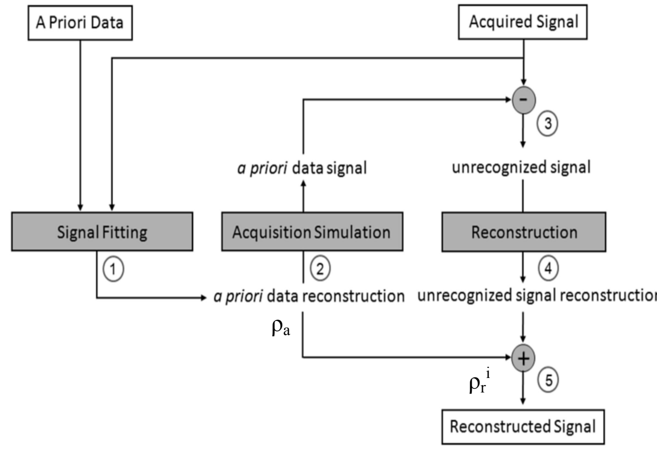

Figure 1. generalized ABSINTHE reconstruction framework. **Step1:** target object reconstruction is approximated with a-priori data so as to minimize number of non matching voxels. **Step2:** a priori object is undersampled **Step3:** subtraction of a priori signal from acquired signal **Step4:** unrecognized signal reconstruction via PI or CS technique **Step5:** a priori and unrecognized signal reconstructions are summed.

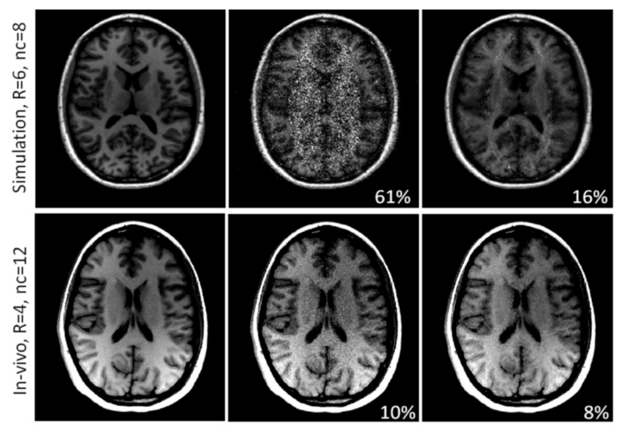

Figure 2. Simulated (top) and *in vivo* (bottom) computations of the original image (left) for SENSE (2[nd] column) and ABSINTHE applied to SENSE (3rd column). Normalized RMSE (%) shown as inset. Image intensity scaled to accentuate artifacts.

Method: A database of 120 T1-weighted, axial, 256x256 images provided by the Open Access Series of Imaging Studies (OASIS) were used as the training set. The background of each image was set to zero using a mask computed from Otsu's segmentation method (6), then multiplied pixelwise with the sensitivity profiles of a numerical 8-channel coil. A Fourier transform was applied to yield a dictionary of multi-coil k-space signals. A 121[st] brain was selected and its multi-channel k-space was calculated with the same numerical coil sensitivities. Normally distributed complex noise was then added to this k-space, with a standard deviation chosen so as to simulate an average SNR of 50 across the brain. Estimates of the coil sensitivity maps were obtained using the adaptive method (3) from these noisy fully-sampled images. Retrospective Cartesian parallel imaging acquisitions were then mimicked by undersampling the signal by reduction factors (R) of R=6 in the phase encoding direction. SENSE and two iterations of ABSINTHE reconstructions were performed in Matlab, as well as an R=1 SENSE reconstruction to serve as ground truth to compute the normalized root mean square error (RMSE) of each reconstruction. The presented ABSINTHE method was additionally tested with *in vivo* data: 35 head volumes were acquired using a standard 12 channel head coil on a 1.5T Siemens Avanto scanner (Siemens Medical Solutions, Erlangen, Germany), using a standard T1-weighted Spin Echo sequence (TR=500ms TE=9.5ms, 2 avg, Slice=5mm, 19 slices, 5mm x 0.3 gap). A central slice from the 35th brain was selected to perform a retrospective simulation, undersampled by a factor of R=4. The training set was made up of similar slices selected from the other 34 brains. SENSE and two iterations of ABSINTHE reconstructions are compared, and the R=1 SENSE reconstruction serves as ground truth to compute the normalized RMSE.

Results: Results are displayed in fig2. ABSINTHE achieves consistently reduced RMSE for both simulation (top) and *in vivo* (bottom) experiments. The diffuse reconstruction noise of SENSE appears to be significantly reduced in ABSINTHE. This type of result can be expected from the reconstruction of a sparse signal added to a noise-free a priori reconstruction.

Discussion: The regularization term introduced for signal reconstruction allows the enforcement of sparsity with fast analytical computation of solutions with improved conditioning and reconstruction. The smaller reduction of RMSE with ABSINTHE in the *in vivo* case compared to the simulations can be attributed to the smaller *in vivo* training set.

Reference: (1) Pierre EY, et al. ISMRM 2010; p.2875 (2)Pierre EY, et al. ISMRM 2011; p2896: (3) Walsh DO et al. MRM 2000 (4) Lin FH et al., MRM 2004;51:559-567 (5)Tsao et al, MRM 2003;50:1031-42 (6) Otsu N. Automatica 1975;11:285-296 **Acknowledgements:** the OASIS database, funding from Siemens Medical Solutions and NIH 1RO1HL094557 and 5K99EB011527.