Determination of the Point Spread Function for Compressed Sensing Reconstruction

Iulius Dragonu[1], Guobin Li[1], Jeff Snyder[1], Jürgen Hennig[1], and Maxim Zaitsev[1]
[1]*Dept. of Radiology, Medical Physics, University Medical Center Freiburg, Freiburg, Germany*

Introduction: Compressed Sensing (CS) is a technique that allows accelerating data acquisition in the presence of sparse or compressible signals [1]. This is accomplished by using a pseudo-random undersampling in the phase-encoding (PE) direction. The Point Spread Function (PSF) is a fundamental tool allowing the evaluation of the quality of reconstruction and the spatial resolution of images. Previous work extended the concept of PSF approximation to non-linear and non-stationary imaging systems [2]. Due to the non-linearity and non-stationary proprieties of the CS algorithms, the PSF has different values in all imaging points. Robson *et al*. proposed the technique of accurately determine the PSF using constant time imaging [3], which proved valuable in echo-planar acquisitions for correcting geometric distortions [3, 4]. In this work we propose a technique of evaluating the PSF of the CS reconstruction based on the acquisition pattern proposed by Robson *et al*. [3].

Theory: For the particular case of the echo-planar sequence, the PSF mapping method is based on the traditional blipped EPI readout with the phase encoding (PE) prewinder replaced by a gradient table [3, 4]. In the case of CS reconstruction this sampling pattern can be simulated using an acquisition of *k*-space data respecting the Nyquist criteria with double the resolution in the PE direction compared to the desired image resolution. Subsequently, the same pseudo-random undersampling pattern is applied on all generated *k*-space data as shown in Fig. 1. This technique assigns to each slice a 3D volume with axes directions of readout, PE (accelerated) and PSF encoding (respecting Nyquist criteria). The first step of the reconstruction is a Fourier transform in the readout direction followed by a non-linear CS reconstruction in the PE direction. This was accomplished by using the following non-linear minimization:

$$\text{argmin}(\|FT_u(x)-y\|^2_2 + \lambda_1\|TV(x)\|_1 + \lambda_2\|\Psi(x)\|_1)$$

where FT_u represents the undersampled Fourier transform, x the reconstructed image, y the acquired *k*-space data, TV the total variation, Ψ the wavelet transform, and λ_1 and λ_2 are regularization parameters to compromise between data consistency and sparsity. This reconstruction was repeated for each step in the PSF encoding dimension. The CS reconstructed image of interest can be extracted from this volume at $k_{PSF} = 0$. The PSF data in each point of the image can be obtained after performing a Fourier transform in the PSF encoding dimension. Note that the reference full-Fourier encoded image can be calculated by Fourier transformation of the raw *k*-space data in the readout and PSF direction and extraction of the image corresponding to $k_{PE} = 0$. The relation between the CS reconstructed image and the full-Fourier encoded image can be visualized using the PSF function as shown in Fig. 1b.

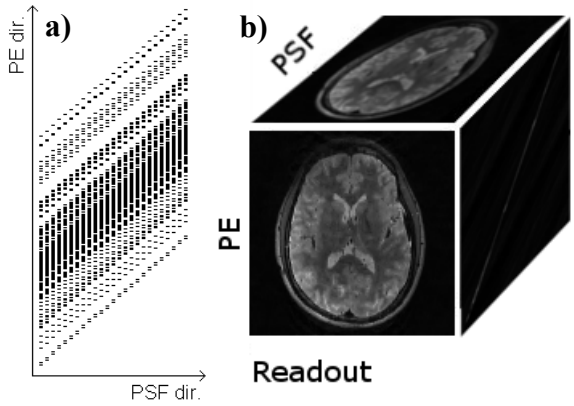

Fig. 1: a) Acquisition pattern for the CS-PSF method. **b)** Correspondence between CS reconstructed image (vertical plane) and full Fourier image horizontal plane using the PSF.

Material and Methods: Full *k*-space acquisitions of a volunteer brain were measured using a TSE sequence with the following parameters TR = 4000 ms, TE = 329 ms FOV = 220×220 mm^2 and acquisition matrix 224×224 resulting in an in plane voxel size of 0.5×0.5 mm^2. Subsequently, for the purpose of the PSF estimation the final resolutions of the CS and full Fourier reconstructed images were reduced by a factor of two.

Results/Discussions: The CS reconstructed images obtained with increasing accelerating factors (2.5 and 3) can be visualized in figures 2(a) and (b), together with the corresponding PSF in the pixel with coordinates (100, 75), (Fig.2(c) and (d), respectively). In order to illustrate resolution loss with increasing acceleration factors the region of interest at the peak of the PSF was displayed separately for three different acceleration factors (R = 2, 2.5 and 3), As shown, peak broadening with increasing acceleration factors is evident.

Conclusion: The image quality for non-linear, non-stationary reconstruction techniques such as CS can be evaluated with the proposed PSF reconstruction technique. The determination of PSF is useful for CS reconstruction to identify the loss of resolution as well as residual back-folding artefacts. In order to estimate the PSF in each point a number of CS reconstructions equal to the number of PE steps was necessary in contrast with the technique proposed by Wech *et. al*. which required ($N_{readout} \times N_{PE}$) reconstructions. However, the technique proposed here requires acquisition of k-space data respecting Nyquist criteria and with the double resolution in the PE encoding dimension. It is to be noted that for non-linear reconstruction methods like CS, the PSF is not shift invariant, however surrogate averaged parameters may be derived based on the PSF data.

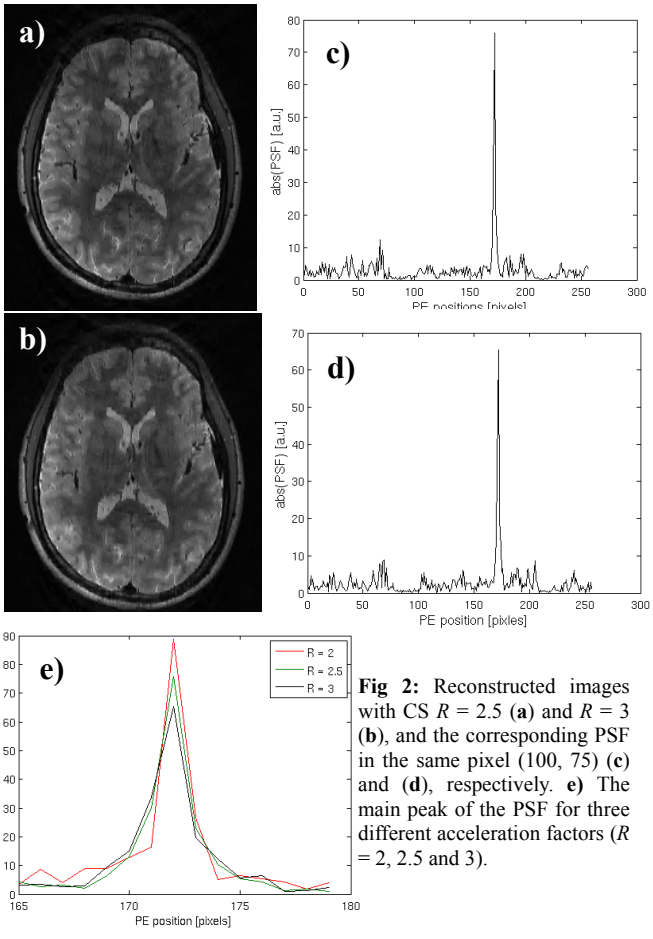

Fig 2: Reconstructed images with CS R = 2.5 (**a**) and R = 3 (**b**), and the corresponding PSF in the same pixel (100, 75) (**c**) and (**d**), respectively. **e)** The main peak of the PSF for three different acceleration factors (R = 2, 2.5 and 3).

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