Increasing Sparsity in Compressed Sensing MRI by Exponent of Wavelet Coefficients

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INTRODUCTION: Compressed sensing (CS) was introduced to the field of magnetic resonance imaging (MRI) in recent years as a promising method to significantly reduce scan time [1-3]. The performance of CS depends on the sparsity of the image in the sparse domain, such as wavelet transform (WT) domain. In this report, we proposed a method to increase the sparsity of CS MRI by taking exponent of wavelet transform (EXP-WT) normalized to the range [0 1]. The method was tested on a digital phantom and in vivo MRI data, and the results show that EXP-WT can improve the quality of the reconstructed image or significantly speed up the reconstruction.

METHODS: *Sparsification by Exponential Operation of Wavelet Coefficient* Mathematically, the reconstruction of CS MRI is obtained by solving the following constrained optimization problem:

$$\min \left\|\Psi x\right\|_1, s.t. \left\|F_u x - y\right\|_2 < \varepsilon \qquad (1)$$

where x denotes the signal to be estimated or the image to be reconstructed, ψ denotes the sparse transform matrix, which, in the present case, transforms the x in the image domain to the wavelet domain, F_u denotes the undersampled Fourier transform, corresponding to the *k*-space undersampling scheme, y denotes the measured *k*-space data from the MRI scanner, and ε controls the fidelity of the reconstruction to the measured data. WT is conventionally used for sparse transform [1] as the wavelet coefficients are sparser than the original signal as shown in Fig. 1 a-b. We found that one could further sparsify the coefficients after normalizing wavelet coefficients to the range of [0 1], and then taking exponential operation n times (Fig. 1 c). Here n is an empirical parameter taken as 6 in this report. The conventional WT and the proposed EXP –WT are formulated in Eq. (2).

$$\Psi_{WT} = WT(x); \quad \Psi_{EXP-WT} = \left[\exp\left(WT(x)\right)\right]_n \qquad (2)$$

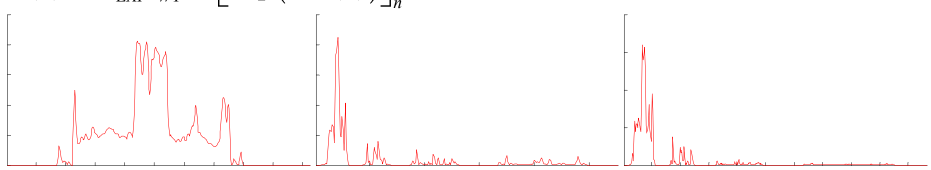

Fig. 1 Original 1D Signal (L), and its WT (M) and EXP-WT Coefficients (R).

Evaluation of the Method We carried out digital phantom and in vivo experiments to evaluate the EXP-WT method. First we undersampled the Shepp-Logan phantom data by a factor of 5, with the variable density pattern [1]. We utilized the bior4.4 wavelet at 5th scale level for wavelet transform. We recovered the phantom image from the 5x undersampled data with 50 iteration steps or without limitation on iteration steps. In the in vivo experiment of T2 weighted brain MRI, we used the bior4.4 wavelet at scale 3 for wavelet transform. The original data matrix was 512 x 512 [4]. We resampled the data with an undersampling rate of 8. We reconstructed the MRI images within 50 iterative steps or unlimited iteration steps. We compared the performances of WT and EXP-WT methods in terms of the qualities of the reconstructed images and the computation times, with or without step limitation, respectively. All the programs were developed in-house using Matlab2011b, and were run on an Intel Core 2, 3 GHz processor with 2GB RAM.

RESULTS AND DISCUSSION: The exponential wavelet operator resulted in a darker sparsity image, as compared to the conventional wavelet operator (Fig. 2), which indicates that the EXP-WT method can greatly increase the sparsity of the image in the sparsity domains. Both the WT- and EXP-WT-based methods achieved nearly the same final result when recovering the images with no limitation on the iteration steps in solving Eq. 1 (Fig. 3). However, the EXP-WT-based method reached better recovery than the WT-based method when the maximal steps are limited, from the view of the noisy background and blurred texture of the recovery of WT method (Fig. 4). A remarkable advantage of the EXP-WT method is that it cost much less time than the WT method, especially when the size of the image was large (Fig. 5). This gives the EXP-WT method practical values in the clinical application of CS-MRI, where both fast data acquisition and rapid image reconstruction are desirable.

CONCLUSION The proposed EXP-WT method can increase the sparsity of the CS MRI image in the wavelet domain, which allows a faster or better reconstruction of the MRI image.

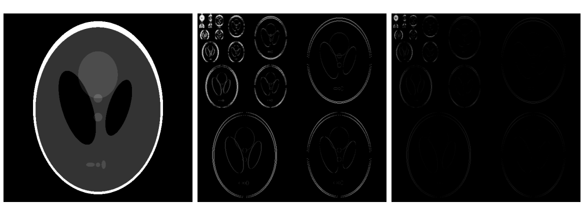

Fig. 2 Original Axial Shepp-Logan Phantom (L), and the WT (M) and EXP-WT Coefficients (R).

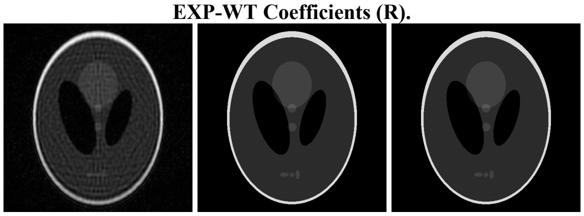

Fig. 3 Recovery with unlimited steps by Zero Filling w/dc (L), WT (M), and EXP-WT(R).

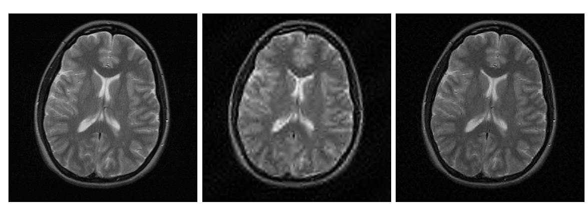

Fig. 4 Original T2 Brain Image (L), and the 8x undersampled CSMRI reconstructed by WT (M) and EXP-WT (R), within 50 iterative steps.

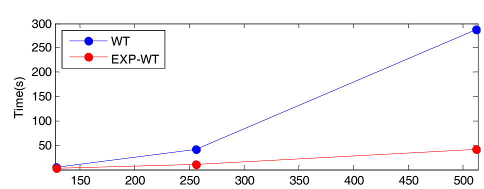

Fig. 5 Comparison of Computation Time

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