Statistical Wavelet Structure Based MRI Compressed Sensing Reconstruction Using a Hidden Markov Tree Model

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Introduction: Compressed Sensing (CS) [1, 2] is an acceleration technique that enables reduction in the sampling and recently applied in MRI [3] based on the sparsity of medical images. Conventional CS recovery techniques are based on simplistic sparsity of signal, which uses uniform L1-norm penalty regardless of whether the coefficients contribute to significant information for pathological diagnosis. This results in reconstruction errors, like blurring edges. In this study, we propose a new algorithm using Hidden Markov Tree model [4] to extract statistical structural information in wavelet domain, such that the sparsity is constrained selectively. In this way important coefficients are enhanced and artifacts are further reduced. Experimental results on phantom simulation and *in-vivo* show more preserved details and better artifact reduction.

Methods:

I. Model Based Compressed Sensing: Conventional CS algorithms approximate MRI images based on simplistic sparse assumption and use a uniform L1-norm penalty for all coefficients [1~3] without considering whether they contribute to fine details in image domain or wavelet domain. Several new studies [5, 6] demonstrated that there are significant performance gains by exploiting realistic models beyond simplistic sparsity.

II. Modeling Wavelet Structure: MRI images have sparse wavelet expansions and significant wavelet coefficients exhibit properties that can be modeled by wavelet quad-tree [7] (Figs. 1.a, 1.b): ***Persistence Property*** (significance persists across the scales), ***Scale-dependence*** (persistence becomes stronger to finer scale) *and* ***Decaying Magnitudes*** (magnitude drops exponentially to finer scale). It should be noted that the wavelet domain structure cannot be defined just by values since they obey mixture distribution and threshing cannot differentiate noise with small but important signal. Thus, statistical methods and models are required.

III. Hidden Markov Tree (HMT) Model: We used a Hidden Markov Tree to model the probability density function of each wavelet coefficient as a Gaussian mixture density [8] with a hidden binary state S_n, which in our application indicates whether coefficients are ***'Negligible'*** ($\boldsymbol{S_n = N}$) or ***'Significant'*** ($\boldsymbol{S_n = S}$). The properties of 2D wavelet quad-tree are captured by a Hidden Markov Quad-Tree Model in which the Gaussian mixture distribution and Transition Matrices between states (See Figs. 1.c, 1.d) are functions of several parameters including the scale $\boldsymbol{J}$ in wavelet-tree.

IV. Regulating CS Reconstruction with Model-based Structure:

We estimated HMT parameters $\widehat{\Theta}(\boldsymbol{\theta}) = \{\boldsymbol{p_1}^{\boldsymbol{N}}, \boldsymbol{p_1}^{\boldsymbol{S}}, \boldsymbol{\alpha_N}, \boldsymbol{\alpha_S}, \boldsymbol{\gamma_N}, \boldsymbol{\gamma_S}, \boldsymbol{C_{\sigma N}}, \boldsymbol{C_{\sigma S}}, \boldsymbol{C_{AN}}, \boldsymbol{C_{AS}}\}$ and the probability of hidden states using Expectation-Maximization (EM) algorithm [7, 9] and a Tree-Viterbi algorithm, which are efficient and enjoy just linear complexity. Then we employed statistical matrices $\widehat{\boldsymbol{W}}_{\boldsymbol{n}}$ based on state probabilities of being ***'Significant'*** to further regulate iterative optimization and statistically penalize coefficients that are '***Negligible'*** (small or isolated) to wavelet structure. *(*$\boldsymbol{\Psi}$*: wavelet operator.* $\boldsymbol{\mathcal{F}_u}$*: partial Fourier transform.* $\boldsymbol{S_n}$*: HMT hidden states. Parameters* $\widehat{\boldsymbol{\Theta}}$ *are computed from measured k-space* $\boldsymbol{y}$ *and reconstructed image* $\boldsymbol{m}$*).*

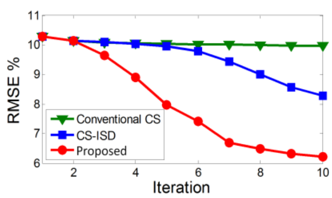
$$\widehat{\boldsymbol{W}}_{\boldsymbol{n}}^{(i)} \propto \boldsymbol{p}(\boldsymbol{S_n} = \boldsymbol{S} | \boldsymbol{\Psi}\widehat{\boldsymbol{m}}^{\boldsymbol{i-1}}, \widehat{\Theta}\,(\boldsymbol{\Psi}\widehat{\boldsymbol{m}}^{\boldsymbol{i-1}}))^{-1}$$

$$\widehat{\boldsymbol{m}}^{(i)} = \boldsymbol{argmin} \left\| \widehat{\boldsymbol{W}}_{\boldsymbol{n}}^{(i)} \boldsymbol{\Psi m} \right\|_{\boldsymbol{1}}, \qquad \boldsymbol{s.t.} \left\| \boldsymbol{\mathcal{F}_u m} - \boldsymbol{y} \right\|_2 < \boldsymbol{\varepsilon}$$

Results:

We tested our algorithm both on Shepp-Logan phantom and *in-vivo* data. Phantom results show our algorithm helps to remove artifacts and preserve edges well (Figs 2). The *in-vivo* brain images from a healthy subject were acquired using a 3-T scanner (GE Healthcare, Waukesha, WI, USA) with T1-flair sequence (TR=2.559s, TE=6.356ms, matrix=256x256, FOV=220x220mm).The data is reconstructed with **4 fold** acceleration along phase-encoding direction, using our algorithm, conventional algorithm (CS) [3] and another algorithm considering wavelet supports based on coefficient values with iterative hard thresh (CS-ISD) [10] (Figs 3) All sharing parameters are the same. RMSE plot(Figs 4) shows the proposed algorithm results in faster and better reconstruction.

Conclusion: A model-based CS algorithm is proposed for MRI reconstruction. The statistical structure of wavelet coefficients are estimated using Hidden Markov Tree Model and sparsity penalty is optimized using structural matrices. Better reconstruction is achieved with fewer iterations, but no more computational complexity or more parameter tuning. This can be further applied to functional and abdominal MRI for which both faster acquisition and fine details are favored. More study is also in progress on how to further improve structure estimation and statistical matrices.



References: [1] Cande`s et al. IEEE TIT 2006; 52(2):489-509 [2] Donoho, IEEE TIT 2006; 52(4):1289-306 [3] Lustig et al. MRM 2007; 58(6):1182-95 [4] Romberg et al. IEEE TIP 2001; 10(7): 1056-68 [5] Baraniuk et al. IEEE TIT 2010; 56(4): 1982-2001 [6] He et al. IEEE TSP 2009; 57(9): 3488-97 [7] Crouse et al. IEEE TSP 1998; 46(4): 886-902 [8] Duarte et al. IEEE ICAASP 2008:5137-40 [9] Dempster et al. JRSS 1977; 39(1):1-38 [10] Liang et al. ISMRM2011

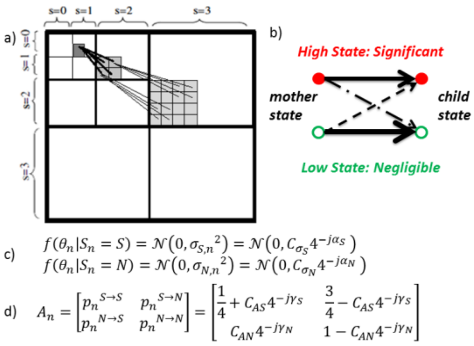


Figure 1: Wavelet quad-tree structure and HMT model. **a:** In wavelet tree, each coefficient is connected to 4 child coefficients. **b:** Two hidden states and their transition are modeled by HMT. **c:** coefficients obey Gaussian mixture density with hidden states. **d:** transition matrices of HMT.

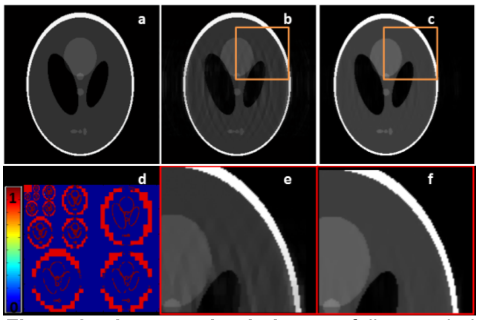


Figure 2: phantom simulations. **a:** fully sampled phantom image, **b-c:** 4-fold reconstruction after 10 iterations, using conventional CS and proposed algorithm. **d:** statistical wavelet structure $\boldsymbol{p(S_n = S)}$. **e-f:** zoom-in views

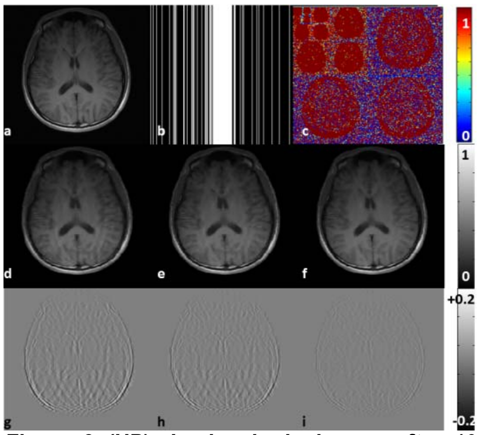


Figure 3 (UP): In-vivo brain image, after 10 iterations. **a**: fully sampled. **b**: 4-fold sampling pattern. **c**:computedstatistical wavelet structure. **d-f**: reconstruction results after 10 iterations, with conventional CS, CS-ISD, CS-HMT. **g-i**: errors.

Figure 4 (UPLEFT): Quantitative comparison. RMSE of 4-fold CS recovery for brain image, with conventional CS, CS-ISD & proposed algorithm.