Coherence Regularization for Compressed Sensing MRI Reconstruction with a Nonlocal Operator

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Introduction: Compressed sensing (CS) [1-2] is a fast growing technique used in MRI because it shortens the imaging time by highly undersampling k-space. Total-Variation (TV) [3] is used in the conventional CS reconstruction to require the finite-differences. However, the TV method usually smoothes noise at the cost of blurring or losing fine details, like edges. In our previous work, we developed a method called coherence regularization with a nonlocal operator [4,5], or CORNOL in abbreviation which can penalize noise while preserving detail structure. In this study we combined CORNOL with CS reconstruction to attempt to solve the problems that CS with TV method encounters. Experimental results on both phantom simulation and in-vivo data show the feasibility of the proposed method.

Theory: The basic assumption of the CORNOL is Coherence Prior. It assumes the intensity changes between correlated pixels are smooth and the same structure in the image is correlated. One equation that provides the flexibility for structure detection is the classic diffusion equation for adaptive image smoothing. $\partial_i \mathrm{u}(\vec{r}) = \mathrm{div}(\Phi(\mathrm{u},\vec{r}) \cdot (\nabla \mathrm{u}(\vec{r})))$ (Definition of Φ and u is seen as follows). By using this equation in a discrete image, the structure of an image can also be extracted. Nonlocal mean filter (NMF) has a robust influence on the image filtering field for its structure- preserving property. The purpose of this filter is to denoise by using the average of a pixel's surrounding to stand for the point. The following nonlocal operator was adopted to characterize image structures in this study: $\Phi_{ij} \frac{1}{w_{ij}} \exp(-\frac{\|p(i)-p(j)\|_2}{\sigma^2})$.The reconstruction of conventional CS is obtained by solving the following constrained optimization problem by CG algorithm [1-2]: $\min(\|\Psi \mathrm{m}\|_1 + \alpha \mathrm{TV})$, $\mathrm{s.t.}\ \|\Psi \mathrm{m}\|_2 < \varepsilon$ .Here we applied CORNOL in the above equation and get CORNOL+CS method. The reconstruction image is thus obtained by solving the following constrained optimization problem. $\min\left(\|\Psi \mathrm{m}\|_1 + \alpha \sum_{j\in\Omega} \langle \overrightarrow{\Phi_i}, |\nabla u_i|^2 \rangle\right)$. $\mathrm{s.t.}\ \|\Psi \mathrm{m}\|_2 < \varepsilon$ .The parameter α is determined by the characteristics of the image, which makes a tradeoff between $\|\Psi \mathrm{m}\|_1$ and $\sum_{j\in\Omega} \langle \overrightarrow{\Phi_i}, |\nabla u_i|^2 \rangle$. $\overrightarrow{\Phi_i}$ is a vector of the discrete values which represent the diffusion coefficient at pixel i. $\nabla u_i$is a vector which means generalized gradient at pixel i. $\langle . \rangle$ is the inner product. When the reduction factor is high and the convergence of the function is hard to achieve, TV is also applied with CORNOL+CS, called improved CORNOL+CS. The reconstruction image is thus obtained by solving the following constrained optimization problem. $\min\left(\|\Psi \mathrm{m}\|_1 + \alpha \sum_{j\in\Omega} \langle \overrightarrow{\Phi_i}, |\nabla u_i|^2 \rangle + \beta \mathrm{TV}\right)$. $\mathrm{s.t.}\ \|\Psi \mathrm{m}\|_2 < \varepsilon$. The parameter β is determined by the reduction factor and characteristics of the image.

Methods: The phantom is a Shepp-logan phantom made by simulated data acquisition matrix=256×256. The in vivo brain data were acquired using a 3-T scanner (GE Healthcare).T1-flair sequence was used to acquire the axial brain images from healthy volunteers. The parameters of the T1-flair sequence were: TR=2.559sec, TE=6.356ms, inversion time=960ms, field of view=220mm×220mm,acquisition matrix=256×256.

Results: Figure1 shows the intermediate steps of using CORNOL. The effectiveness of the method is presented. Figure2 presents apparent difference of applying the conventional CS and improved CORNOL+CS. The total conventional CS iteration time is about 332s. And the iteration time on improved CORNOL+CS iteration is about 548s. The RMSE of the conventional CS and improved CORNO+CS is 0.1014, and 0.0220, respectively. Figure3 shows an in vivo brain image which applied conventional CS and CORNOL+CS. The total conventional CS iteration time is about 235s. And the iteration time on CORNOL+CS iteration is about 224s. The RMSE of the conventional CS and CORNO+CS is 0.0415, and 0.0302, respectively.

Conclusion: We proposed to use CORNOL in CS reconstruction. The results show that CORNOL with CS is able to detect and preserve coherence structures and filter out the noise at the same time. Better reconstruction is achieved in comparison with the original CS In order to further improve the performance of the proposed method; optimizing regularization parameter α is needed

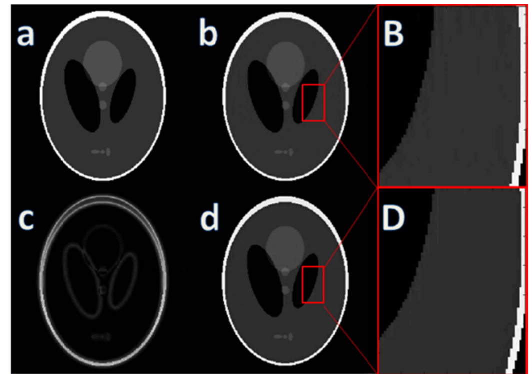

Figure1: Phantom image with reduction factor of 2 at 6th iteration which illustrate the processing and effectiveness of CORNOL. a: original image. b: image with noise and artifact before the CORNOL processing. c: intermediate product of CORNOL, reveals local structures for the Shape-Login phantom image. d: image after the implement of CORNOL, the structure is enhanced, the noise and artifact are penalized.

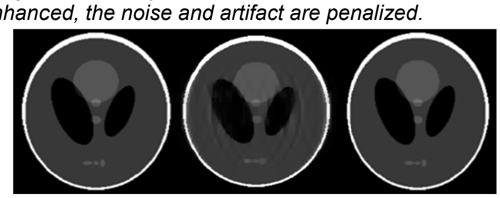

Figure2: Phantom image with reduction factor of 4. Left: the original image. Middle: 15 times of conventional CS *iteration and the differentia. Right: 8 times of improved CORNOL+CS iteration and the difference.*

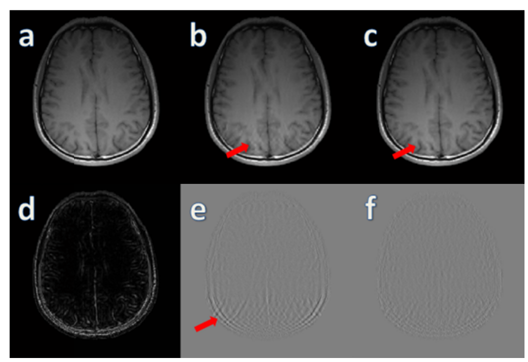

Figure3: in-vivo image with reduction factor of 2 which is used to show the improvement of the method combined CS and CORNOL. a: the original image. b: 10 times of conventional CS *iteration .c: 6 times of CORNOL+CS iterations d: intermediate product of CORNOL, reveals local structures for the brain. e: the difference between a and b. f: the difference between a and c*

References;

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