Improving rank constrained reconstructions using prior information with reordering

Ganesh Adluru[1], Liyong Chen[1, 2], David Feinberg[2], Jeffrey Anderson[1], Edward V.R. DiBella[1]
[1]Radiology, University of Utah, Salt Lake City, UT, United States; [2]Advanced MRI Technologies, Sebastopol, CA, United States

Introduction: Recent developments in matrix completion theory [1, 2] have generated interest in translation to dynamic MR imaging acceleration from undersampled k-t space data [3, 4]. By exploiting the fact that a matrix of dynamic images is highly correlated and has low rank, promising results on dynamic breast imaging [3] and cardiac perfusion imaging [4] have been reported. Here we propose a reordering method that extends these rank constrained reconstructions to incorporate prior image information. Similar ideas to improve reconstructions in the context of compressed sensing have been explored [5-7]. Promising results of the proposed method are presented on undersampled multi-image data from a multi-image diffusion imaging acquisition that could significantly benefit from reduced scan times.

Methods: If *D* is the multi-image undersampled data acquired from a scanner, a rank penalty term can be used as a constraint to resolve the aliasing artifacts as

$$\min_M\big(rank(M)\big)\,;\, s.t. \|E(M)-D\|_2^2 < \epsilon \qquad (1)$$

In (1), *M* is a *m*x*n* matrix of reconstructed images in which each column represents a vectorized complex image from the multi-image acquisition. *m* is the total number of pixels in one image and *n* is the number of images in the acquisition. The operator *E* transforms *M* to match the acquired data *D*. The non-convex rank constraint (L0 norm of singular values) in (1) can be relaxed to a convex nuclear norm constraint (L1 norm of the singular values) $\|M\|_*$in practice without significant loss in the reconstruction quality.

However, when the underlying true image matrix does not satisfy the low rank/nuclear norm constraint, prior information can be injected into the matrix to lower its rank leading to a better justification of the rank constraint. Reordering a matrix so that each of its columns is monotonic significantly lowers its nuclear norm. Figure 1 shows a plot of singular values of a complex image matrix from a fully sampled 64 direction diffusion imaging dataset corresponding to a single coil. When the real and imaginary parts of each column are independently sorted to make them monotonic, the nuclear norm of the reordered matrix is significantly reduced. Hence instead of directly applying the rank penalty term on the image estimate matrix *M*, we propose to reorder each column the matrix using a prior estimate of true signal and perform undersampled data reconstruction as

$$min_M(\|O(M)\|_*);\;\; s.t. \|E(M)-D\|_2^2 < \epsilon \qquad (2)$$

In (2) *O* is an operator that reorders real and imaginary parts of each column of *M* independently using the sorting order from pixel intensities of a prior. Here we use a Spatio-Temporally Constrained Reconstruction (STCR) with a 'temporal' (along diffusion encoding direction) and a spatial total variation constraints [8] as a prior for reordering.

We test the proposed method on a fully sampled brain diffusion imaging dataset from a stroke patient. A b=0 image and 64 encoding directions (b=800) for 36 slices were acquired using a Simultaneous Image Refocusing (SIR) sequence with a SIR factor of two [9] on a Siemens 1.5T scanner with an EPI readout and TR=5.7 s, TE=138 ms, pixel size=1.8x1.8 mm^2, slice thickness=2.5mm. Overlapping slices were separated and corresponding fully sampled k-space data for each slice was undersampled offline in a variable density random fashion by a factor of three. Low rank reconstruction with no reordering was performed in a Projection Onto Convex Sets (POCS) framework. One step of fidelity term update was alternated with minimization of the nuclear norm until convergence was reached. Singular Value Thresholding (SVT) [10,11] was used for minimizing the nuclear norm, singular values below a certain empirically determined threshold were zeroed while those above the threshold were kept. For the reordered rank reconstruction in (2) the ordering was predetermined as the sorting order of pixel intensities of real and imaginary parts from the STCR reconstruction. Reconstruction in a POCS framework was done as described earlier except that SVT was applied on the reordered matrix. Individually reconstructed coils were combined in a sum-of-squares fashion.

Results: Figure 2 shows the results of low rank reconstruction without and with reordering. The rank constraint reduces the undersampling artifacts but does not match the overall sharpness of fully sampled image. Reordering significantly improves overall image quality as seen in the difference image with fewer structures as compared to that without reordering. Root Mean Squared error in the reconstruction with reordering was lower by 14% as compared to that with no reordering.

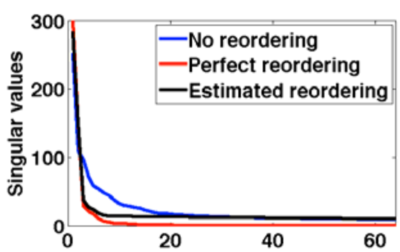

Figure 1. Plots of singular values of a complex image matrix formed from a 64 direction diffusion imaging dataset. The L1 norm of the singular values was reduced by 62% with perfect reordering and by 20% with reordering estimated using the STCR prior.

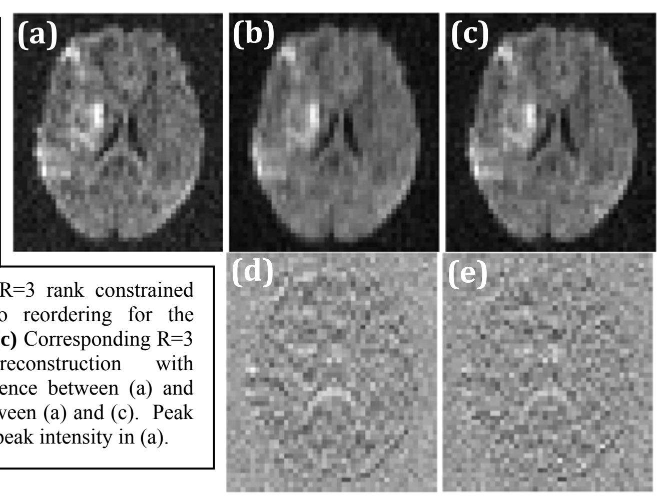

Figure 2: Results of rank constrained reconstructions with reordering. **(a)** Fully sampled data with IFT reconstruction of an axial slice of a diffusion encoding direction.

(b) Variable density R=3 rank constrained reconstruction with no reordering for the corresponding image. **(c)** Corresponding R=3 rank constrained reconstruction with reordering. **(d)** Difference between (a) and (b). **(e)** Difference between (a) and (c). Peak difference is ~20% the peak intensity in (a).

Discussion and Conclusion: In practice it is not possible to find the perfect reordering as obtained using true data. However, taking advantages of different types of tradeoffs in the existing quiver of undersampled reconstruction methods, hybrid prior images can be formed and used in the reordering rank penalty framework. Here we used only STCR images as prior to the rank constrained reconstruction and found improvements. Reordering offers a simple yet promising way to incorporate prior information into rank constrained reconstructions that can offer improved quality.

References: [1] Candes et al. Found. Comput. Math. 9:717-772, 2009. [2] Recht et al. SIAM Rev 52:471-501,2010. [3] Haldar et al. Proc. IEEE ISBI, 716-719, 2010. [4] Goud et al. IEEE TMI 30:1042-54, 2011. [5] Adluru et al. Int J Biomed Imaging, 2008:341684, 2008. [6] Wu et al. Magn Reson Med. 65:83-95, 2011. [7] Ramirez et al. IEEE TMI 30:1566-76, 2011. [8] Adluru et al. J Magn Reson Imaging, 29:466-473,2009. [9] Feinberg et al. Magn Reson Med 48:1-5, 2002. [10] Cai et al. SIAM J on Optimization, 20:1956-1982, 2010. [11] Candes et al. http://svt.caltech.edu/code.html, 2011.