# Optimal Combination of High Frequency Sub-band Compressed Sensing and Parallel Imaging: Consideration of Local and Global Characteristics of k-space

Suhyung Park[1], and Jaeseok Park[1]
[1]*Department of Brain and Cognitive Engineering, Korea University, Seoul, Seoul, Korea, Republic of*

**Introduction:** Fast imaging techniques, which combine compressed sensing (CS) with parallel imaging (PI), have been introduced, demonstrating promising results of speeding up data acquisition and improving spatial and temporal resolution [1-2]. Among them, a combination approach [3], which reconstructs missing signals in k-space locally and separately using CS (high-frequency reconstruction) and PI (low-frequency reconstruction), was recently developed. Despite the fact that this approach exploits local characteristics of k-space optimally for CS and PI, since CS and PI reconstructions were performed in the locally segmented k-space, it potentially produces signal discontinuity particularly in the boundaries between high frequency sub-band regions as well as between low frequency (PI reconstruction) and high frequency (CS reconstruction) regions. To resolve signal discontinuity problems in k-space and further suppress artifacts and noises, in this work we develop a novel reconstruction algorithm that takes into account both local and global characteristics of k-space and thus optimally combines high frequency sub-band CS and PI.

**Methods:** In the high frequency sub-band CS with PI algorithm (HiSub CS) three-level variable density sampling pattern was applied: 1) fully sampled calibration region, 2) uniformly under-sampled region for PI, 3) incoherent under-sampled region for CS. Uniformly under-sampled data in the central k-space were filled using interpolation kernel obtained from calibration region, and each section of k-space corresponding to the high frequency sub-bands in the wavelet (Ψ) domain were reconstructed by solving the local independent L1 minimization problem: $\min\|w_i\|_1,\ \mathrm{s.t.}\ \|\Phi w_i - m_i\|^2,\ \ i = LH, HL, HH\ (\mathbf{1})$, where $w_i$ denotes coefficients of high frequency subbands (i=LH, HL, HH) in the wavelet domain, Φ is the Fourier transform, and$m_i$ is the measured data in the sub-section of k-space corresponding to the high frequency subbands (i=LH, HL, HH) in wavelet domain. Since only local characteristic of k-space is considered, signal discontinuity occurs in the boundaries between high frequency sub-sections as well as between low (PI reconstruction) and high (CS reconstruction) frequency regions, potentially resulting in artifacts and amplified noises. To resolve the signal discontinuity problems, the proposed reconstruction algorithm consists of the following two steps (Fig. 1): 1) L1 minimization with data constraint that considers both local high frequency sub-sections and global high frequency section in k-space (CS initialization) and 2) L2 minimization with iterative refinement of low and high frequency signals using PI (PI refinement).First, after the three-level variable density sampling, missing signals in the central k-space are filled using PI. High frequency k-space sampled in a pseudo-random fashion is inverse Fourier transformed, which will be used as a global data constraint for L1 minimization. The entire k-space that undergoes only PI reconstruction (low frequency region) is transformed to a wavelet domain and will be used as a local constraint for L1 minimization. CS initialization of high frequency signals is performed by the following L1 minimization with both local and global data constraints: $\min\|w_i\|_1,\ \mathrm{s.t.}\ \|[\Phi w_H]_i + \Phi w_i - m_i\|^2,\ \ i = H, LH, HL, HH\ (\mathbf{2})$. Second, the CS-Initialized high-frequency k-space is then combined with the PI-initialized low frequency signals in k-space (Init k-Space). Assuming that convolution kernel in PI is invariant over the entire k-space including both CS and PI reconstructed regions, the concept of self-consistency [4] is adopted with the init k-space as a baseline to synthesize and refine CS and PI data over the entire k-space using:$\min_x\|Gx - x\|_2^2 + \lambda\|x - x_0\|_2^2(\mathbf{3})$, where x is the desired reconstructed k-space, $x_0$ is the init k-space, G is the data interpolation kernel, and λ is a weight parameter to balance between the init k-space and self-consistency. Eq.3 can be solved using a steepest descent algorithm with a step size μ: $x_{n+1} = x_n - \mu[(G - I)^*(G - I)x_n + \lambda(x - x_0)](\mathbf{4})$, where n is the iteration number. With increasing iteration number, signal discontinuity existing in between CS and PI data gradually decreases, potentially enhancing data accuracy of both low and high frequency.

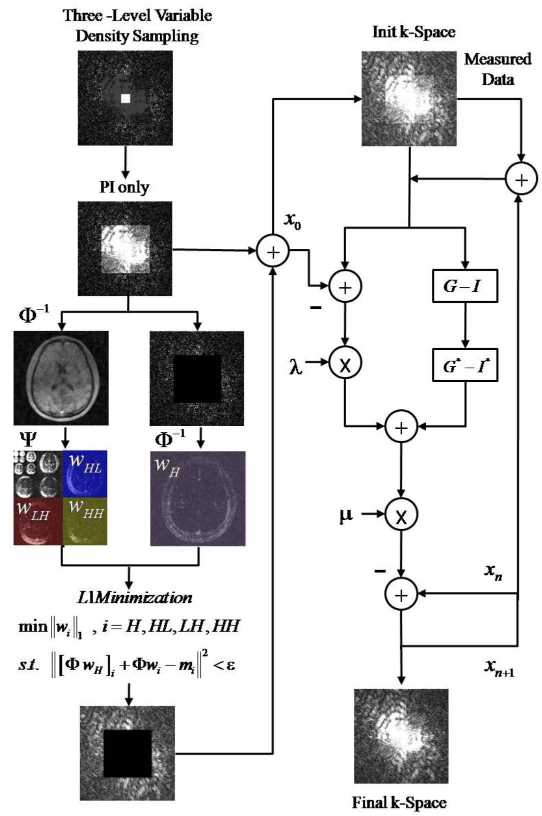


***Figure 1:*** *An overall schematic of the proposed reconstruction algorithm*

**Results:** A brain in vivo data is acquired in a volunteer at 3T whole-body scanner (MAGNETOM Trio, Siemens Medical Solutions, Erlangen, Germany) using spoiled gradient echo imaging with following parameters: FA, 70°;TE, 2ms; TR, 22ms FOV, 220mm x220mm; in-plane acquisition matrix, 256x256; slice thickness, 4mm. A twelve-channel head coil is used for signal reception. A set of fully acquired data is decimated using an under-sampling factor of$2 \times 2$ ($k_y \times k_z$) for PI and 6.2 for CS. Three images are reconstructed using only PI, HiSub CS, and the proposed technique, for comparison. PI-only yields aliasing and ringing artifacts. HiSub CS decreases ringing artifacts but still yields residual aliasing artifacts. The proposed technique successfully eliminates signal discontinuity in k-space and thus outperforms the others in reducing artifacts and noises

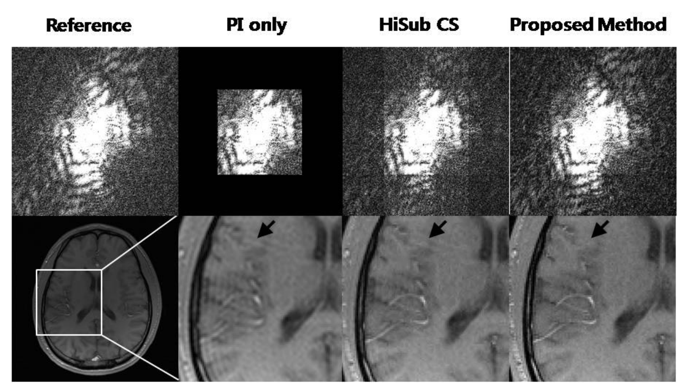


**Figure 2:** *Upper Row: Reconstructed k-spaces, Bottom Row: Reconstructed Images*

**Conclusion and Discussion:** we successfully demonstrated a novel reconstruction algorithm that optimally combines high frequency sub-band CS and PI, taking into account both local and global characteristics of k-space. The proposed technique is a promising and efficient solution particularly for high resolution imaging. **Reference:** [1] Griswold et al., MRM 47:1202-10 (2002), [2] Lustig et al., MRM 58:1182-95 (2007), [3] K. Sung et al., ISMRM 2011, p.70, [4] Lustig et al., MRM 64:457-471 (2010).

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