CS-SENSE or Denoised SENSE: The Influence of Irregular Sampling in l1 Regularized SENSE Reconstruction

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Introduction:

Parallel imaging (PI) is a well established method for scan acceleration in MRI [1,2]. Since the introduction of SparseMRI [3] the combination of compressed sensing (CS) and PI has been of great interest to further accelerate MRI scans [4-7]. While CS exploits incoherent sampling and signal sparsity to solve an ill-posed inverse problem, sensitivity encoding (SENSE) uses the prior knowledge of coil sensitivities to solve an over-determined problem. However, the SENSE encoding matrix is usually ill-conditioned, which leads to noise amplification in the reconstructed images [1]. The conditioning of the encoding matrix decreases with increasing SENSE reduction factor limiting the achievable scan acceleration. To counteract the noise amplification in SENSE, regularization methods accounting for noise correlation and image support are used [8,9]. Inspired by the CS literature, ℓ_1-regularization was also proposed to improve SENSE reconstruction [10,11]. In the case of regular undersampling, the ℓ_1-regularization performs denoising and will be called ℓ_1-denoised SENSE within this work.

Both CS-SENSE and ℓ_1-denoised SENSE can be formulated as an ℓ_1-regularized least squares problem, the main difference being the incoherent sampling in CS-SENSE. The term ℓ_1-regularized SENSE will be used here, generalizing for both regular and irregular sampling. In this work, we investigate the influence of the sampling pattern on the convergence behaviour of ℓ_1-regularized SENSE reconstruction at different reduction factors. In other words, we try to answer the question what improvement can CS-SENSE provide over ℓ_1-denoised SENSE.

Methods:

3D brain data were acquired on a 1.5T clinical scanner (Philips Healthcare, Best, The Netherlands) using an 8 channel head coil and a TFE sequence (TE/TR=4.5/12.5, FOV=240x240x176mm^3, 1mm isotropic voxel). The sensitivity maps and image support estimation were obtained from a standard low resolution SENSE reference scan. Full data sets were acquired and retrospectively undersampled. The following sampling patterns were considered for reduction factors of 4 and 9: 2D regular undersampling (reduction factors 2x2 and 3x3), uniform density Poisson disk (PD) sampling, and variable density PD sampling with high undersampling in the k-space periphery.

Image reconstruction was performed using Nesterov's optimal gradient scheme [12] adapted to solve the optimization problem:

$f(x) = ||F_u Sx - y||_2^2 + \lambda_1 ||R^{-1}x||_2^2 + \lambda_2 ||\Psi x||_1$, where x is the image to be reconstructed, y is the measured data, F_u is the undersampled Fourier transform, S is the coil sensitivity matrix, the operator R^{-1} selects the image regions not included the estimated image support, and Ψ is a wavelet transform. In addition, standard regularized non-iterative SENSE reconstruction was performed on the regularly undersampled data.

Results:

Reconstruction results for different undersampling factors are shown in Fig.1. As expected, ℓ_1-regularization reduces the noise amplification compared to standard SENSE. At a reduction factor of 4 (Fig.1a), the ℓ_1-regularized SENSE reconstruction converges to a solution with the same ℓ_2 error for all considered sampling patterns. The convergence curves for regular sampling and uniform density PD sampling are very similar, while variable density PD sampling results in faster convergence.

At a reduction factor of 9 (Fig.1b), regular undersampling results in severe noise amplification. Uniform density PD sampling leads to a reduced reconstruction error. Altogether 300 iterations were performed; however, there was no further improvement in the reconstruction error after about 100 iterations for these two sampling patterns. Variable density PD sampling provides the best results both in convergence speed and image quality.

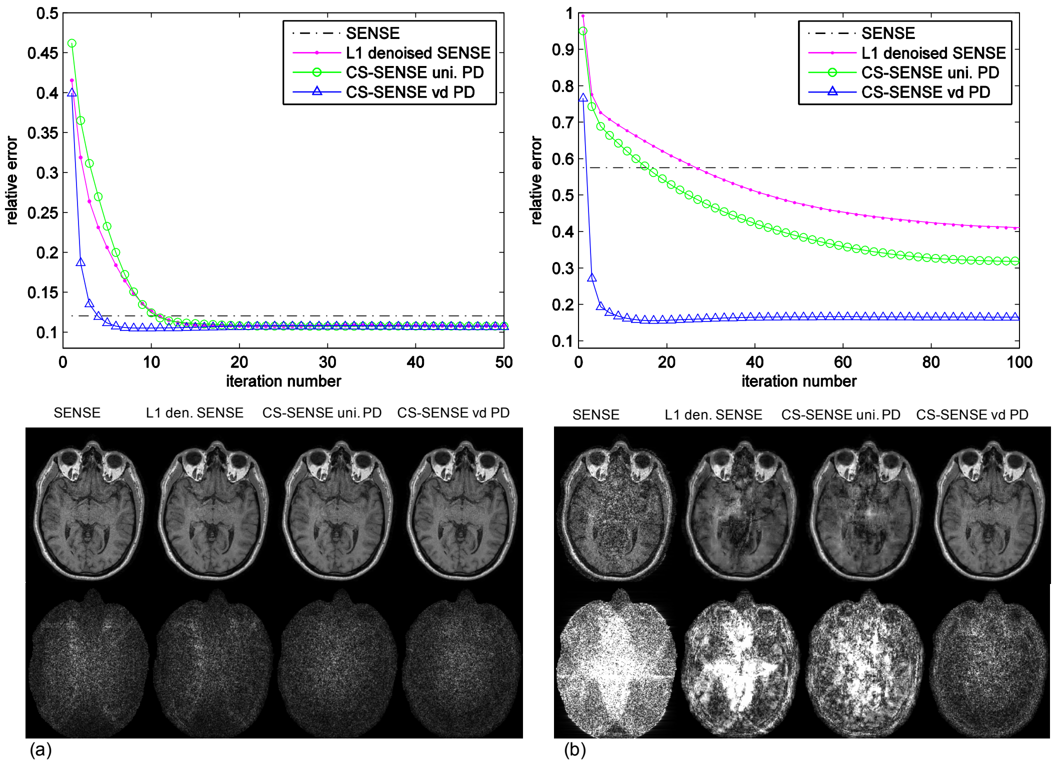

Fig.1. Convergence curves (top) and image reconstruction results (bottom) for (a) reduction factor 4 and (b) reduction factor of 9. Top: Normalized RMS error with respect to the fully sampled image as a function of the number of iterations is given for ℓ_1-denoised SENSE and CS-SENSE with uniform and variable density Poisson disk sampling. The reconstruction error of non-iterative SENSE is given for reference. Bottom: Reconstructed images and difference images (with respect to the fully sampled image) after (a) 50 iterations (b) 300 iterations.

Discussion and Conclusion:

At low to moderate reduction factors, ℓ_1-regularized SENSE is an overdetermined inverse problem. Irregular sampling results in more homogeneous noise amplification and faster convergence in case of the variable density sampling. However, the reconstruction converges to a solution with the same total reconstruction error for all sampling patterns. At high reduction factors, the reconstruction problem becomes ill-posed and CS theory applies. In this case, irregular sampling leads to a reduced reconstruction error. The improvement in the reconstruction error is especially noticeable for variable density sampling, because it better exploits the sparsity of the high frequency data. Variable density sampling also provides the best convergence rate, which is almost unchanged for different reduction factors.

With respect to the achievable image quality and the small number of iterations needed, variable density incoherent sampling shows to be a promising candidate for clinical application of ℓ_1-regularized SENSE reconstruction.

References: [1] Pruessmann K et al, MRM 1999, 42:952-962 [2] Griswold M et al, MRM 2002, 47:1202-1210 [3] Lustig M et al, MRM 2007, 1182-95; [4] Wu B *et al*, ISMRM 2008; #1480 [5] King K et al, ISMRM 2008; #1488. [6] Liu B et al, ISMRM 2008; #3154 [7] Liang et al. MRM 2009, 62: 1574-84 [8]Lin F et al, MRM 2004 51: 559-567 [9] Eggers H et al ISMRM 2008 # 12 [10]Uecker M et al., MRM 2008, 60: 674-82 [11] Liu B et al, MRM 2009, 61: 145-52 [12] Nesterov Y. Soviet Math Dokl 1983; 372-76.