Regularized reconstruction using redundant Haar wavelets: A means to achieve high under-sampling factors in non-contrast-enhanced 4D MRA

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Introduction: Recently, a novel time-resolved data acquisition method was proposed in [1] for non-contrast-enhanced (NCE) 4D intracranial MR angiography (MRA). It uses two ECG-triggered CINE-like b-SSFP acquisitions of multiple 3D phases after selective and non-selective inversion, respectively. By subtraction of the two acquisitions, the stationary signal is suppressed, while the difference between the signals of inflowing non-inverted and inverted blood provides dynamic vascular information. For high spatial and/or temporal resolution, a long scan time would be required, and partially parallel imaging is effective to the reduction of the total scan time by acquiring a reduced number of *k*-space samples. As a *k*-space sampling scheme, the spiral phyllotaxis pattern [2] can not only achieve uniform dense *k*-space sampling but also significantly reduce the eddy current effects [3]. In this work, a method is proposed that exploits the spatial sensitivity information of the multiple coil elements and makes use of a regularization based on 4D redundant Haar wavelet transformation for incorporating both spatial and temporal structures in order to reconstruct NCE 4D MRA from the under-sampled *k*-space data acquired with the simulated spiral phyllotaxis pattern.

Methods: Let X be a 4-dimensional tensor representing our target 4D difference image, where X_i denotes the 3-dimensional difference image at temporal phase i. Let Y_i^j denote the difference of the under-sampled *k*-space data of the two acquisitions at temporal phase i by the j-th coil, where the sampling scheme used at temporal phase i is represented by the operator P_i. Let F denote the 3-dimensional Fourier transform, and S_i^j be the 3-dimensional coil sensitivity map for the j-th coil at temporal phase i. Let W denote the 4-dimensional redundant Haar wavelet operator applied to a 4D tensor, and D be a 4D weight tensor. Let $\odot$ denote the Hadamard (or componentwise) product between two tensors. In our proposed method, the 4D difference image X was reconstructed by minimizing the following function:

$$\min_X \frac{1}{2}\sum_{i=1}^{t}\sum_{j=1}^{c} ||Y_i^j - P_i(F(S_i^j \odot X_i))||_F^2 + \lambda||D \odot (W(X))||_1, \qquad (1)$$

where the first term is the data fidelity depicting the discrepancy between the measured *k*-space data and the one that is estimated from X, and the second term is the penalty term incorporating the piecewise constant structures in both spatial and temporal directions. For the sampling operator P_i, the spiral phyllotaxis pattern was used for retrospectively generating the readouts in the line-partition plane, with nearest neighbor interpolation onto a Cartesian grid, as illustrated in Fig. 1. To incorporate the spatial and temporal structures, an L_1 regularization based on the 4D redundant Haar wavelet was used. Redundant Haar wavelets have been proven effective for two-dimensional MRI reconstruction [4]. In the proposed method, higher weights were imposed to the temporal direction, as the adjacent temporal phases are very close to each other. In addition, zero weights were imposed to the difference (and the mean) between the first phase and the last phase, as they turn out to be quite different. The problem (1) was solved by FISTA [5].

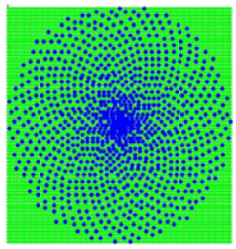

Fig. 1 Illustration of the under-sampled k-space samples in the line-partition plane by the spiral phyllotaxis pattern.

The NCE 4D MRA data were acquired in healthy volunteers on a 3.0T clinical MR scanner (MAGNETOM Trio, A Tim System, Siemens Healthcare, Germany). Imaging parameters included TR/TE=4.15/1.75 ms, field of view=220×165 mm^2, matrix=240×180, 8 coils elements, 72 slices, 0.8 mm thickness, 16 temporal phases, temporal resolution=49.8 ms, BW=627 HZ/Pixel, and flip angle evolving from 15º to 45º. Partial Fourier was applied to acquire 6/8 of data in the phase-encoding, and a slice resolution of 50% was used. In line-partition plane, the sampled readouts occupied a rectangle area of size 132×36. The total acquisition time for this fully sampled data set was 15.8 minutes.

To generate the under-sampled *k*-space data, an AND operation was applied between the acquired data and the simulated spiral phyllotaxis pattern, resulting an acceleration factor of 13.7. For the spiral phyllotaxis pattern, the Fibonacci number was set to 8; and to generate diverse sampling locations for different temporal phases, the last readout of the previous temporal phase was rotated by a golden angle to obtain the angle of the first readout of the next temporal phase. For the weight tensor D in (1), the weights for the spatial directions were set to 1, the weights for the temporal direction to 5, and the weights associated with the difference (and mean) between the first phase and the last phase to 0. A temporal phase independent coil sensitivity map for all temporal phases was used, which was estimated with the approach discussed in [6] using the 24 reference lines acquired for the last temporal phase.

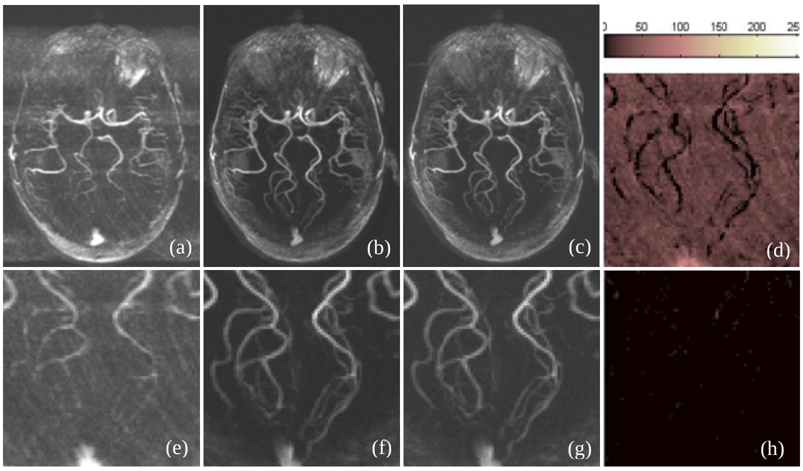

Figure 2 MIP images: images reconstructed by (a) SoS_{under}, (b) 4D Wavelet & (c) SoS_{scan}; (e)-(g) are the zoomed images of (a)-(c); (d) difference between (e) and (g); (h) difference between (f) and (g); and (h) is displayed using the same color map as (d) .

Results: The proposed 4D Wavelet method was compared with 1) SoS_{under}: sum of squares of the inverse Fourier transform with zero-filling to the under-sampled data obtained by the simulated spiral phyllotaxis pattern, and 2) SoSscan: sum of squares of the inverse Fourier transform with zero-filling to the full acquired dataset. Maximum-intensity projection (MIP) was performed in the transverse plane. In Fig.2, the results of the last temporal phase are shown for a representative volunteer. Fig. 2 (a) illustrates the aliasing artifacts when the Fourier transform was directly applied to the under-sampled data with zero-filling. Fig. 2 (b) gives the results generated by the proposed 4D Wavelet approach, which achieves comparable visual image quality as SoS_{scan} shown in Fig. 2 (c). Zoomed images of Fig. 2 (a-c) are shown in Fig. 2 (e-g), and the difference images are presented in Fig. 2 (d) & (h) using the color map shown in (d). Compared to Fig. 2 (g), the NRMSE's (normalized root-mean-square error for Fig. 2 (e) & (f) are 10.5% and 0.3%, respectively. It can be observed that the proposed approach reconstructs fine details with high accuracy, while aliasing artifacts are efficiently suppressed. Our results on four other NCE 4D MRA data sets give similar results as shown in Fig. 2.

Discussion and Conclusion: NCE 4D MRA proposed in [1] is a promising non-invasive technique for visualization of vascular anatomy. It usually requires a long scan time especially when high spatial and/or temporal resolution is desired. In this work, the (weighted) 4D redundant Haar wavelet was proven an effective tool for reconstruction from under-sampled *k*-space data acquired by the simulated spiral phyllotaxis pattern. Our results show that excellent results can be achieved with an acceleration rate of even 13.7. Our ongoing work includes accelerating further with the usage of more coil channels and modifying the sequence for direct acquisition of only the under-sampled *k*-space data for further volunteer studies.

References:

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