Artifact-free resolution enhancement using a shearlet sparsity prior

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Introduction

Existing compressed sensing (CS) MRI reconstruction techniques can handle a wide variety of undersampled k-space acquisitions corrupted with noise. On top of that, these methods can also be employed for resolution enhancement applications. Very often, clinicians will 'zoom in' on an MR image, where the zooming is actually implicitly performed by bilinear, bicubic or sinc interpolation. These algorithms are computationally simple, but give rise to artifacts. As such, in this abstract, we propose a CS MRI reconstruction algorithm, which uses a sparsifying transformation, tailored to the problem of reconstructing an enlarged image from undersampled k-space data. We will also discuss how we accelerate this algorithm by optimally choosing the free parameters in the augmented Lagrangian reconstruction formulation.

Method

In CS MRI, reconstruction an image x is reconstructed by solving an optimization problem, using k-space data y, as:

$$\hat{x} = \arg\min_{x} \|Fx - y\|_2^2 + \lambda \|Sx\|_1 \qquad (1)$$

Where S is a sparsifying transform and F is the k-space sampling operator. In [1], it was shown how using the shearlet transform, because of its optimal sparsifying property, results in improved reconstruction performance. One drawback with the shearlet transform is its redundancy, which has received little attention. The implementation of [2] focuses on reducing this redundancy, but this comes at the cost of using bandlimited filters, as the filter bank is implemented in the Fourier space.

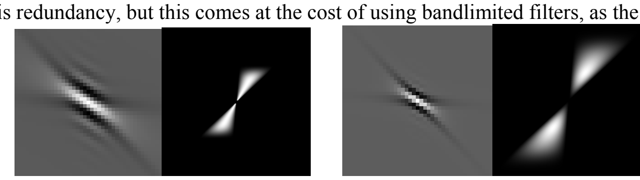


Figure 1: Image of one mid-scale shearlet and its spectal support for the low redundancy shearlet transform (left) and the proposed smooth radial shearlet transform (right)

Bandlimitedness eliminates the option of having spatially compact shearlet basis functions, which manifests as ringing, as can be clearly seen in Figure 1. To avoid this ringing in the reconstructed image, we build the shearlet transform with very smooth radial filters. In our implementation, the radial filters were implemented using a pseudo radial version of the db2 wavelet, which gives rise to the shearlet shown on the right in figure 1.

In [1], it was proposed to solve the optimization problem using an augmented Lagrangian optimization technique, as in [3]. This introduces a new parameter μ, and now the optimization algorithm is performed iteratively by repeating three steps:

$$\begin{cases} x_{i+1} = \arg\min_x \|Fx - y\|_2^2 + \mu \|d_i - Sx - b_i\|_2^2 \\ d_{i+1} = \arg\min_d \lambda \|d\|_1 + \mu \|d - Sx_{i+1} - b_i\|_2^2 \\ b_{i+1} = b_i + Sx_{i+1} - d_{i+1} \end{cases} \qquad (2)$$

The result of this optimization does not depend on the choice of μ, but the convergence speed can be greatly affected by it. We propose the following method for making an optimal choice for μ. When assuming $F^H F \approx 1$, which is true for well-posed, correctly regridded reconstruction, and assuming that at convergence, the soft thresholding does not modify the signal in a significant way, it is possible to consider the first step in (2) as a infinite impulse response (IIR) filter such that:

$$\begin{cases} d_{i+1} = \mathrm{ST}\left(Sx_{i+1} + b_i | \frac{\lambda}{\mu}\right) \approx Sx_{i+1} + b_i \\ x_{i+1} \approx \frac{\mu}{1+\mu} x_i + \frac{1}{1+\mu} y \end{cases}$$

Under these assumptions, analysis of the transient effect of the IIR shows that μ should be chosen as low as possible in order to have fast convergence. However, if μ<1, the first assumption, that soft thresholding does not modify the signal in a significant way, no longer holds. This can most easily been seen for the case of simple image denoising, where F and S are the unit matrix. The result of (1) could then be found in one step: soft thresholding of y with threshold λ. Both observations lead us to choosing μ=1 as the parameter for optimal convergence speed.

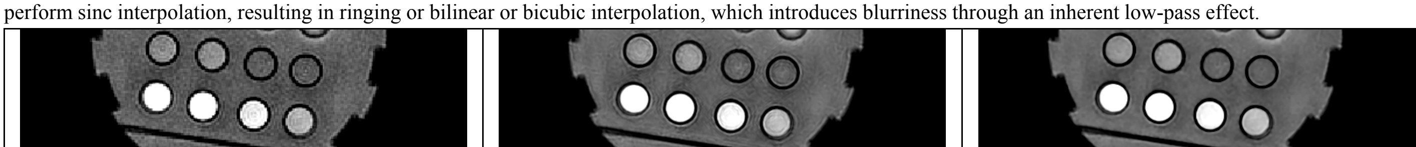
Experiments and conclusion

In a first experiment, convergence speed was evaluated. In this scenario, a well-posed reconstruction experiment was performed and different choices for the parameter μ were evaluated. Further experiments have demonstrated that μ=1 remains a good initial guess for fast convergence, even for ill-posed reconstruction.

Table 1: required number of iterations for convergence in a well-posed reconstruction experiment for different choices of μ

μ	0.01	0.1	0.5	1	2	10	100
# iterations until convergence	>5000	870	180	130	240	1110	>5000

In the second experiment, a phantom was placed in a Siemens Trio scanner and sampled along a fully sampled Nyquist spiral. This data is a textbook example for the ringing artifact that is seen in some MRI modalities. The ringing artifact occurs from sinc interpolation of data with missing frequencies. The proposed algorithm stands out in that it can mitigate the ringing artifact, by increasing resolution (zooming in) without altering the acquired data. Typical image viewing software would either perform sinc interpolation, resulting in ringing or bilinear or bicubic interpolation, which introduces blurriness through an inherent low-pass effect.



Original resolution regridded reconstruction	2x resolution enhancement using the shearlet implementation with bandlimited radial filters	2x resolution enhancement using the shearlet implementation with smooth radial filters

Figure 2: Reconstruction results for spirally acquired MRI data from a phantom.

Results show that ringing is mitigated and that the proposed method can increase the apparent resolution in a non-trivial way, through the use of a sparse image prior.

Bibliography

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