Highly Accelerated Parameter Mapping with Joint Partial Separability and Sparsity Constraints

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INTRODUCTION

MR parametric mapping provides quantitative information of intrinsic tissue properties, which are useful biomarkers. However, the utility of MR parametric mapping has been limited by long scan times. Recently, sparse sampling methods based on the theory of partial separable functions [1] and compressed sensing (CS) [2] have shown great potential for accelerating MR data acquisition. Some of the methods have been extended to parametric mapping [5][6][7]. In this work, we present a new method to accelerate parametric mapping using the PS and sparsity constraints jointly and illustrates its performance in T2-mapping. The proposed method is shown to achieve much more accurate T2 maps than existing methods using a single PS or sparsity constraint with significantly reduced imaging time.

METHODS

Consider T2 mapping as an example, and let $\rho(\mathbf{r}, t_n)$ for $n = 1, 2, \cdots, N$, represents a set of T2-weighted images with variable T2-weightings. With sparse sampling, direct inversion of the ($\mathbf{k}$, $\mathbf{t}$)-space measured data can incur severe artifacts in the reconstructed images and subsequently the T2 map. We have previously proposed reconstruction methods using joint PS and sparsity constraints for accelerating dynamic imaging [3][4]. The same approach can be easily adapted for parametric mapping. Specifically, the PS model assumes $\rho(\mathbf{r}, t_n) = \sum_{l=1}^{L} u_l(\mathbf{r}) v_l(t_n)$, where L is the model order. Considering the Casorati matrix $\mathbf{P}$ formed from samples of $\rho(\mathbf{r}, t_n)$ such that $\mathbf{P}_{m,n} = \rho(\mathbf{r}_m, t_n)$. The PS model can be equivalently expressed as $\mathbf{P} = \mathbf{U}_s\mathbf{V}_t$, where $\mathbf{V}_t$ represents the temporal subspace, and $\mathbf{U}_s$ represents the corresponding the spatial coefficients. Under the PS model, the measured data can be written as $\mathbf{d} = \Omega(\mathbf{F}_s\mathbf{U}_s\mathbf{V}_t) + \xi$, where $\Omega(\cdot)$ is the sparse sampling operator and ξ is the measurement noise. Multiple data acquisition schemes can be designed to determine $\mathbf{V}_t$. In Fig.1, we illustrate one sampling pattern, in which fully sampled central $\mathbf{k}$ space data is used as temporal training data. With $\mathbf{V}_t$ being determined, the reconstruction problem is equivalent to solving $\mathbf{U}_s$. However, directly determining $\mathbf{U}_s$ from measured data can suffer from severe ill-conditioning issue. Imposing spatial sparsity constraints can effectively stabilize the ill-conditioned inverse problem. Specifically, we use a spatial finite difference to regularize the inverse problem of fitting $\mathbf{U}_s$ for the PS model, and the reconstruction problem can be formulated as,

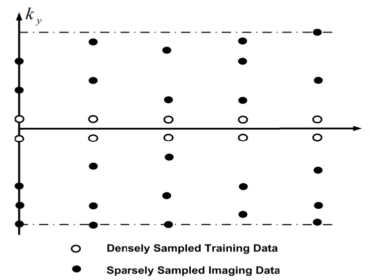

Fig. 1. A sampling pattern example for PS-Sparse, which consists of densely sampled temporal training data, and sparsely sampled imaging data.

$$\hat{\mathbf{U}}_s = \arg\min_{\mathbf{U}_s} \| \mathbf{d} - \Omega(\mathbf{F}_s\mathbf{U}_s\mathbf{V}_t) \|_2^2 + \lambda \mathrm{TV}(\mathbf{U}_s) \tag{1}$$

where $\mathrm{TV}(\cdot)$ represents the spatial total variation regularization. The above convex optimization problem can be efficiently solved by half-quadratic minimization with continuation [3][4]. After $\rho(\mathbf{r}, t_n)$ is obtained, the desired parameter map is obtained using a nonlinear least squares fitting procedure such as VARPRO (variable projection) [8].

RESULTS

The proposed technique, named as PS-Sparse, was validated in a brain T2 study. The brain of a healthy volunteer was scanned at 3T with a spin-echo sequence at 32 different echo times. The imaging parameters are 2s TR, 17.2 ms echo spacing, and 256x208 matrix size. The data set was retrospectively undersampled at an acceleration factor of 10 using the sampling pattern shown in Fig. 1. The down-sampled data set was reconstructed with (1) compressed sensing [5]; (2) partial-separable model [6]; and (3) the proposed PS-Sparse technique. Figure 2 compares the T2 maps estimated from the reconstructions using (1) CS, (2) PS, and (3) the PS-Sparse. The T2 map estimated from fully sampled 32 echoes is used as the reference. As can be seen in Fig. 2 (a), the proposed PS-Sparse produces a T2 map with higher spatial resolution and SNR than the CS and PS counterparts with the CS showing significant spatial blurring and the PS low SNR.

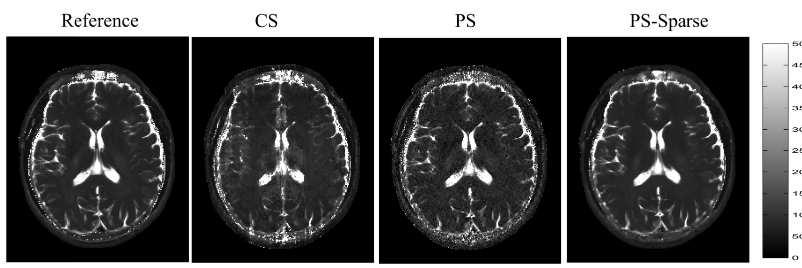

Fig. 2. a) Estimated T2 maps from CS, PS, and PS-Sparse reconstructions at an acceleration of 10, respectively; b) Error maps of estimated T2 maps using Sparse, PS, and PS-Sparse

DISCUSSION

Several sparse sampling-based fast parametric mapping methods have been proposed recently, such as the *k-t* PCA method [6] and REPCOM [7]. The *k-t* PCA is based on the basic PS model and suffers from the ill-conditioning issue illustrated in Fig. 2. REPCOM uses a similar formulation based on joint PS and sparsity constraints with $\mathbf{V}_t$ being estimated from a pre-fixed parametric signal model. The proposed method uses the temporal subspace estimated from physically acquired training data, which can more faithfully capture underlying temporal evolution of relaxation process.

CONCLUSION

A new image reconstruction method based on joint partial separability and sparsity constraints has been proposed to accelerate MR parametric mapping. The method shows superior performance to existing methods using a single PS or sparsity constraint for reconstructing highly undersampled data.

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