Joint RF pulse and gradient design for 2D spatially selective excitation using optimal control and B-spline waveform model: initial experience

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INTRODUCTION

Two-dimensional spatially selective RF pulse design evolved from investigation under the small-tip-angle (STA) assumption for single transmission [1] to general design without such constraints with applications to parallel transmission, using either least squares optimization [2,3] or variational approaches [4]. Most works focused on optimization of the RF waveform given a fixed EPI or spiral gradient trajectory, with some exceptions using joint optimization of both waveforms [5] with the least squares formulation. In this work, we propose a joint RF and gradient waveform design algorithm under the framework of optimal control theory using a cubic B-spline waveform model [6]. Preliminary results with an initial spiral trajectory show that the algorithm can improve the excitation profile over the initial design under the STA assumption, especially for large tip angles. The proposed algorithm can be adapted to design of parallel transmission RF pulses.

METHODS

The effective magnetic field in the Larmor rotating frame can be written as $B_{eff}(\mathbf{r},t)=[s_1(\mathbf{r})b_{1x}(t), s_1(\mathbf{r})b_{1y}(t), \Delta B(\mathbf{r})+\mathbf{G}^T\mathbf{r}]^T = \mathbf{E}(\mathbf{r})\mathbf{u}(t)+\Delta(\mathbf{r})$, where $\mathbf{u}(t)=[b_{1x}(t),b_{1y}(t),g_x(t),g_y(t),s_x(t),s_y(t)]^T$ is the control vector with $b_1(t),g(t),s(t)$ being the B1, gradient, and slew rate waveforms and $\mathbf{E}(\mathbf{r})$ the constant 3 by 6 coefficient matrix. Let $\varphi(M(T))=\|D-M(T)\|^2$ be the cost of the final excitation state. The optimization cost function formulated with the Lagrange multiplier is: $J(\mathbf{u}(t);M_0)=\varphi(M(T))+\int_0^T L(\mathbf{u}(t),t)dt+\int_0^T\sum_r \lambda^T(\mathbf{r},t)[\gamma M(\mathbf{r},t)\times B_{eff}(\mathbf{r},t)-M_t'(\mathbf{r},t)]\,dt$, where $L(\mathbf{u}(t),t)=\frac{1}{2}\int_0^T \alpha_b\|b_1(t)\|^2+\alpha_s\|s(t)\|^2\,dt$ is the regularization term. Define the Hamiltonian: $H(M,\lambda,\mathbf{u},t)=L(\mathbf{u}(t),t)+\sum_r \lambda^T(\mathbf{r},t)[\gamma M(\mathbf{r},t)\times B_{eff}(\mathbf{r},t)-M_t'(\mathbf{r},t)]$. Then with given M,λ trajectories under certain boundary condition [7], the cost is determined by $\int_0^T H(M,\lambda,\mathbf{u},t)\,dt$. Using the formula $a^T(b\times c)=c^T(a\times b)$, we derive $H(M,\lambda,\mathbf{u},t)=L(\mathbf{u}(t),t)+\mathbf{p}^T(t)\mathbf{u}(t)+\varepsilon(t)$, where $\mathbf{p}(t)=\gamma\sum_r \mathbf{E}^T(\mathbf{r})[\lambda(\mathbf{r},t)\times M(\mathbf{r},t)]$, and $\varepsilon(t)=\gamma\sum_r[\lambda(\mathbf{r},t)\times M(\mathbf{r},t)]^T\Delta(\mathbf{r})$. In this work, both the RF and gradient waveforms are modeled using cubic B-spline functions in the form of $f(t)=\sum_{k=0}^N c_k\beta(t/h-k)$. This model turns the optimization against the temporal RF and gradient waveforms (real and imaginary) to against a parameterized vector $\mathbf{c}=[c_{xk},c_{yk},d_{xk},d_{yk}]$, respectively. According to Pontryagin's necessary condition for weak variations [7], at the optimal control $\mathbf{u}^o(t)$ and the corresponding optimal paths $M^o(t),\lambda(t)$, $H_u(M^o,\lambda,\mathbf{u}^o,t)=0,\ \forall t\in[0,T]$. Note that without the B-spline parameterization, one could use the gradient descent algorithm treating the Hamiltonian as an intermediate cost function $\hat{J}(t)=H(M^o,\lambda,\mathbf{u}^o,t)$ whose gradient w.r.t. the control $\mathbf{u}(t)$ is computed for all t. However, due to the fact that the Hamiltonian H is a function of t and the control parameterization $\mathbf{c}$ is not, it is impossible to directly use such a gradient descent algorithm to solve for $\mathbf{u}^o$. We solve this problem by a modified intermediate scalar cost function $\tilde{J}(\mathbf{c})=\int_0^T\|H_u(M^o,\lambda,\mathbf{u}^o(\mathbf{c}),t)\|^2\,dt$, whose gradient descent direction points to the minimization of H_u , $\forall t\in[0,T]$. Since the numerical range of the RF and gradient waveforms can differ quite substantially, we compare the performance of simultaneous optimization against both waveforms to alternative optimization against each individual waveform. In our implementation, initial RF and gradient waveforms are designed using Pauly's approach under the STA assumption [1]. Subsequently the initial pulses are optimized using both the simultaneous and alternative optimization algorithms described above. The STA design cannot be implemented directly in practice since the RF and gradient waveforms do not start/end at zero. Thus it has to be fitted to an implementable waveform. We use cubic B-spline functions to fit to the STA design. In the following, the STA design, the fitted STA design, the proposed simultaneous joint optimization and the alternative joint optimization schemes are denoted as STA, STA-fit, SJD, AJD respectively.

RESULTS AND DISCUSSION

Preliminary results with numerical simulation of the Bloch equation are shown here with the following design parameters: spiral gradient trajectory, field-of-view (FOV) = 20cm, field-of-excitation (FOEx) = 5cm with a disk shape, kmax = 0.5cycles/cm, maximum gradient strength = 4G/cm, maximum slew rate = 15G/cm/ms, and 512 temporal samples on the waveform constructed with 256 B-spline control coefficients. Figure 1 shows the excitation profiles of the STA, STA-fit, SJD and AJD designs of a 90° RF pulse. Note that the STA and STA-fit designs generated significant magnetization along the x axis, which should be zero by design. The SJD design gives improved profile in terms of x-magnetization as well as final cost (see Fig. 2), but contains noticeable ringing artifact both inside and outside the desired excitation region. The AJD design shows significant improvement over the SJD design, although some ringing artifact remains. Figure 2 shows the cost of the final excitation profile computed against the desired excitation pattern for the aforementioned 4 designs for a range of flip angles. It is seen that the proposed AJD generates the best excitation profile and has the smallest error. The phase coherence of the excited region is slightly degraded but this should be well within the tolerable range.

CONCLUSION

We describe a new algorithm for joint RF and gradient waveform design using optimal control and a B-spline signal model for 2D spatially selective RF pulses. The benefit of using B-spline signal model for the waveforms is computation efficiency due to the local support of the B-spline basis. For certain applications, it is also much easier to build smoothness constraints into the waveform design thanks to the intrinsic smooth nature of B-spline functions. We have adapted the optimal control framework by changing the intermediate cost function to accommodate the parameterization of the waveforms. Our initial results show the effectiveness of the proposed algorithm. Also note that the algorithm is easily adapted to design of RF pulses for parallel transmission. The current implementation written in MATLAB is able to complete the optimization of a 90° pulse in about 3 minutes, which we expect will be significantly reduced if the implementation is optimized for speed.

REFERENCES

[1] Pauly et al., JMR, 81(1), 1989; [2] Yip et al, MRM, 54(4),2005; [3] Grissom et al, MRM, 56(3), 2006; [4] Xu et al, MRM, 58(2), 2007; [5] Yip et al, MRM, 58(3), 2007; [6] Unser, IEEE Sig. Proc. Mag., 16(6), 1999; [7] Speyer and Jacobson, Primer on Optimal Control Theory, SIAM 2010

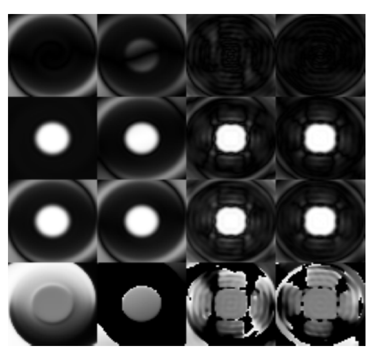

Fig.1. Excitation profiles of different design schemes. From top to bottom: magnitude of x magnetization, magnitude of y magnetization, magnitude of transverse magnetization and phase of transverse magnetization. From left to right: STA, STA-fit, SJD, AJD.

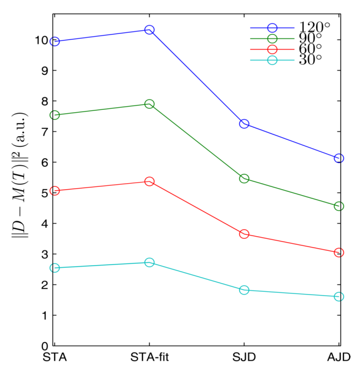

Fig.2. Final cost of different design schemes at flip angles of 120, 90, 60 and 30 degrees.