PARAMETER ESTIMATION OF THE BOLD FMRI MODEL WITHIN A GENERAL PARTICLE FILTER FRAMEWORK

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INTRODUCTION

Neural activity in the brain triggers a chain of physiological activities collectively named as the hemodynamic response. The Blood Oxygenation level Dependent (BOLD) signal is an end result of this collective response. With attempts to model this series of dynamic changes, the first compelling version of the hemodynamic forward model was proposed in [1], namely, the Balloon model. Ever since their introduction these models have been extensively applied to analyse the Functional Magnetic Resonance Imaging (fMRI) data. The hemodynamic parameters associated with the model can give an understanding to functional differences of the regions of the brain and/or between subjects. However, parameter learning with respect to nonlinear model fitting to actual fMRI data is practically difficult and is still an open issue. We present a novel simulation based Bayesian approach for nonlinear model based analysis of the fMRI data.

During the last decade Sequential Monte Carlo (SMC) methods or Particle Filters (PF) have become the state-of-the-art of filtering for nonlinear/non Gaussian models. In the present work we use an intact PF implementation algorithm for joint estimation of states and parameters of the hemodynamic model. There have been previous PF approaches for model based fMRI BOLD signal analysis [2]. However, the main difference in our work is that we consider the static/dynamic parameter estimation within a general filtering framework.

METHODS

We use an extended version of the Balloon model [1], supplemented with a damped oscillator to model the blood flow in [3] for our analysis. The full path of the model we use describes the dynamics of the normalised cerebral blood flow f_t the normalised flow inducing signal s_t, the normalised blood volume v_t and the normalised deoxyhemoglobin content q_t with ordinary differential equations. The BOLD signal y_t as given in [4] is $y_t = V_0\big(a_1(1-q_t) - a_2(1-v_t)\big)$ where the subscript t indicated the time instance. This model can be given in a general form of a continuous time stochastic nonlinear system including uncertainly in the system evolution and additive measurement and instrumental noise. In order to apply in a general filtering framework we use a time discretization of the states using a simple first order Euler-Maruyama scheme.

In the context of Bayesian estimation the solution to an estimation problem is given by the *a posteriori* density $p(x_t|Y_t)$i.e. a posteriori density estimation of a variable at a specific time given all the measurements /observations up to that time point $Y_t = [y_1, y_2, \dots, y_t]$. In the joint estimation this posterior is modified as $p(x_t, \theta|Y_t) \propto p(y_t|x_t, \theta)p(x_t|\theta, Y_{t-1})p(\theta|Y_{t-1})$. PF's approximate this posterior with a set of weighted particles [5]; $\{x_t^i, w_t^i\}$ where $i = 1,2, \dots, N_s$ and N_s is the number of particles. Amongst the different PF algorithms we use a sequential Importance Resampling (SIR) algorithm [5]. To do the joint estimation we use an augmented state vector, and to address the issue of degerancy due to non dynamics of parameters we use a kernel smoothing approach presented in [6].

RESULTS

The validity of the proposed method was evaluated on a synthetically generated data set. For simplicity we only focus on estimating the parameters transit time through the balloon τ_0, time constant of signal decay τ_s and time constant of feedback auto regulatory mechanism τ_f while the others are held constant at mean values reported in [3] and [4]. The simulated data are generated with τ_s=1.25, τ_f=1.667, τ_0=1.2. The system is discretised at 0.1s time intervals (Δt). The process and measurement noise variances are set to 0.01 and 0.001 respectively. We use the values reported in [4] as the priors of the model parameters to initialize the PF and $N_s = 1000$. We use a block design paradigm for the stimulus, and assume that the response or the BOLD signal is captured at 2s time points. Figure 1 shows the parameter estimates and their posterior distributions. The results show accurate estimates of sates and parameters in a very high sampling time of 2s. The final estimates of the parameters are τ_s=1.315, τ_f=1.667, τ_0=1.2783.

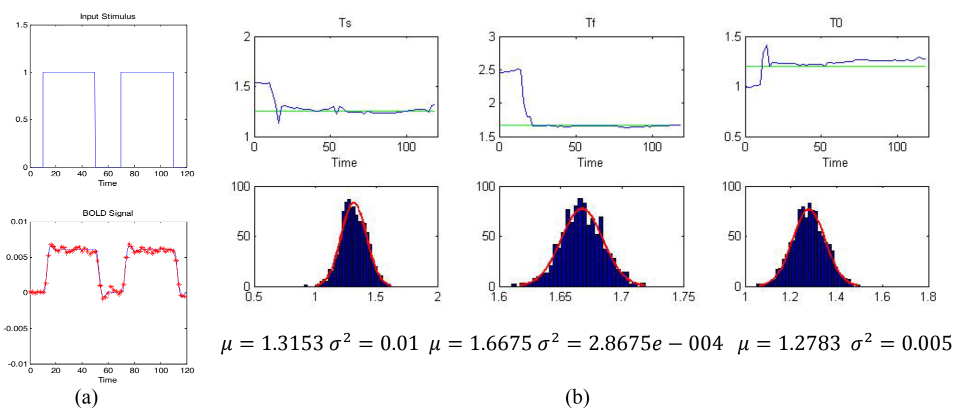

Fig. 1. Parameter Estimates of the BOLD model. (a) Top: The block input, Bottom:Simulated BOLD signal (blue-without noise, red-noisy). (b) Top:Parameter convergence Bottom: Normal approximation to posterior parameter histograms. Time axis is shown in seconds.

CONCLUSION

In this paper, we presented a joint estimation method associated with the PF for estimation of states and parameters of a BOLD hemodynamic model within a general filtering framework. Parameter estimate convergence with low variance of the final estimate, even at a very high sampling time, show promising results to be applied in experimental data analysis. Though convergence takes a relatively long time, with typical experimental durations this must not be problematic. However, one may argue that the Particle filters suffer from the *curse of dimensionality*, with increasing number of parameters in the model. Thus our future work will focus on overcoming these issues with this powerful nonlinear model estimation technique.

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