# Anisotropic error propagation in *q*-ball imaging

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**Introduction:** Several publications in the past have addressed the analysis of anisotropic error propagation in DTI, e.g. the accuracy [1] or precision [2] of fractional anisotropy as a function of fibre orientation with respect to different DW encoding schemes ("response functions" [1]). Recently, model-free diffusion imaging methods that require high angular resolved diffusion imaging (HARDI) schemes have become more popular. *Q*-ball imaging (QBI) is one popular technique which deduces the (fibre) orientation density function, ODF, from a single spherical shell sampled in *q*-space is [3]. Although several works have investigated QBI error propagation (e.g. [4]), to the best knowledge of the authors, none has yet investigated QBI error *anisotropy*. In contrast to DTI, the ODF reconstructed from noise free signals is already biased by methodological approximations and various parameters such as b-value, spherical harmonics expansion order, etc., and not least the encoding scheme used.

**Methods:** In this work, the anisotropy of QBI accuracy without noise as well as QBI accuracy and precision with a typical signal-to-noise ratio (SNR) is compared for several encoding scheme types and for a range of reasonable DW encoding numbers, $N$, up to 240. To avoid angular response function undersampling, 1000 fibre orientations, $x$, uniformly covering one hemisphere are simulated (a two-or-more-fibre configuration would require uniform orientations described by three Euler angles instead of two). Three encoding scheme types, here denoted as "CFmin$N$", "Disco$N$" and "Spiral$N$", with $N$={30,60,80,120,240} repeated {8,4,3,2,1} times are used to generate diffusion signals based on a diffusion tensor model (prolate with tr(D)=2.3·$10^{-3}$ mm$^2$/s and FA=0.8) with a known true ODF$_0$ given a fixed b-value (here, b=1500 s/mm$^2$) [5]. "CFmin$N$" refers to commonly used schemes numerically optimised by minimisation of Coulomb forces between point charges on a sphere [6]. In contrast to the analytical "Spiral$N$" schemes [7], originally not designed for diffusion weighting, the DISCOBALL schemes ("Disco$N$") were designed to be antipodally symmetric [8]. Further common schemes can be derived by Icosahedral tessellation but then only limited numbers are available (6,21,81,321,…) necessitating a separate comparison with non-constant number of excitations (NEX). For the Monte Carlo simulations Rician noise corresponding to SNR=15 in one non-weighted average was added to 10000 signal repetitions. Multiple b=0 weighted signals were simulated according to NEX$_{b>0}$:NEX$_{b=0}$=240:25. An analytical, regularised QBI reconstruction is applied using a 6$^{th}$ order spherical harmonics expansion [5]. A moderate and commonly used regularisation value of λ=0.006 was chosen. In this work, the anisotropy of the mean and standard deviation response functions of the symmetrised Kullback-Leibler divergence (KL) are analysed – a parameter which quantifies the information difference between estimated ODF and ground truth ODF$_0$ [3,4]. For each fibre orientation, $x$, and Monte Carlo repetition, $n$, KL is estimated by sampling both ODFs at 500 uniformly distributed directions, $r_i$, according to Eq. 1:

$$KL(x,n) \quad = \quad 0.5 \cdot \sum_{i=1}^{500} ODF(r_i)\log(ODF(r_i)/ODF_0(r_i)) + ODF_0(r_i)\log(ODF_0(r_i)/ODF(r_i)) \tag{1}$$

Note that the ODF sampling scheme (to be distinguished from the encoding scheme) generally also introduces a directional bias which is here minimised by applying a CFmin500 scheme with a reasonably large number of sampling directions >>$N$. In analogy to the (generalised) fractional anisotropy from (QBI) DTI Eq. 2 defines the anisotropy of a general response function, $f(x)$, such as the KL mean and standard deviation over the Monte Carlo repetitions:

$$An(f) \quad = \quad std(f)/rms(f) \quad = \quad 1/\sqrt{1+|mean(f)|^2/std(f)^2} \tag{2}$$

The weighted average and weighted standard deviation over $f(x)$ are inserted into Eq. 2 to counterbalance residual non-uniformity in the simulated fibre orientations defined by a CFmin1000 scheme (weights proportional to the spherical Voronoi diagram areas).

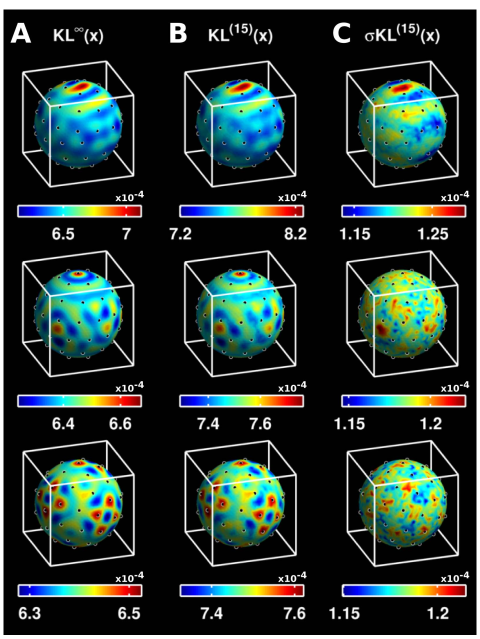


***Fig. 1:*** *Typical response functions of KL accuracy without (A) and with noise (B) and KL precision (C). From top to bottom: 8*(Spiral30, Disco30, CFmin30).*

**Results and Discussion:** Fig. 1 shows examples of KL response functions with SNR=∞ (A, "KL$^\infty$") and with noise according to SNR=15 (B, "KL$^{(15)}$") and the related standard deviation response functions (C, "σKL$^{(15)}$"). Fig. 2 displays the mean and the anisotropy of the response functions for all simulated numbers of directions. All mean and anisotropy curves monotonically decrease though converge at different rates – the DISCOBALL curves almost as quickly as the CFmin curves clearly in contrast to the Spiral curves. The limit depends on various parameters kept constant in this comparison, e.g. the ODF sampling scheme or NEX. Although not shown here, similar response functions are obtained for the generalised fractional anisotropy. Additionally, Fig. 3 demonstrates, on the example of KL$^\infty$ response functions, the destructive impact of the unequal triangle sizes accompanying Icosahedral tessellation on QBI error anisotropy.

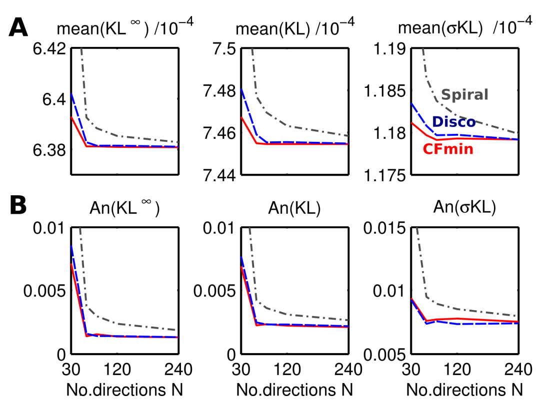


***Fig. 2:*** *Mean KL (A) and KL anisotropy (B) for b=1500 s/mm$^2$ as a function of unique diffusion directions for two deterministic and one optimised scheme type (CFmin).*

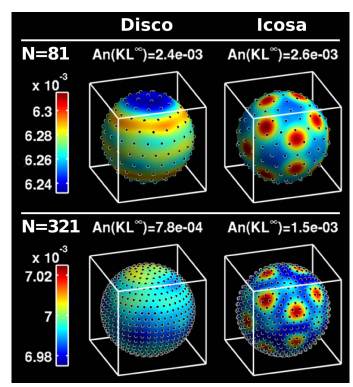


***Fig. 3:*** *The KL response functions of Icosahedral tessellation schemes (right) show more anisotropy than the DISCOBALL schemes (left).*

**Conclusions:** Three practical main conclusions can be drawn: (I) QBI errors are anisotropic; the anisotropy depends on the encoding scheme used (cf. Fig 1) and converges to a minimum for increasing numbers of directions (analog to DTI, cf. Fig 2); (II) despite the large number of directions compared to DTI, a proper, i.e. highly uniform encoding scheme choice is crucial for HARDI acquisitions (cf. Fig 2), note that the Icosahedral schemes often used may be disadvantageous with regard to this (cf. Fig 3); (III) the inherent (noise free) QBI bias allows for a comparably easy way of qualitative error anisotropy assessment (cf. Fig 2 left and middle column), which is useful in cases where Monte Carlo simulations are too challenging. This allows, for example, for a discussion of QBI error anisotropy with two-or-more-fibre configurations.

**References:** [1] Jones, DK, *MRM* 2004; [2] Skare, S et al., *JMR* 2000; [3] Tuch, DS, *MRM* 2004; [4] Zhan, L et al., *Neuroimag* 2009; [5] Descoteaux, M et al., *MRM* 2007; [6] Jones, DK et al., *MRM* 1999; [7] Wong,S et al., *MRM* 1994; [8] Stirnberg, R et al., *Proc. Intl. Soc. Magn Reson Med* 2009