Performance Evaluation of a Non-linear Finite-Element Model for Assessing Yield Strength from in Vivo 3D MR Images of Trabecular Bone

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Introduction: Osteoporosis is a common bone disorder that leads to increased risk of fracture. To assess osteoporotic fracture risk high-resolution MR (μMR) or CT (μCT) image-based micro-finite element (μFE) modeling has been used to estimate bone mechanical properties [1,2]. Linear μFE models are widely utilized to calculate bone elastic parameters. However, linear models as predictors of bone strength assume a relation between bone elastic properties and failure strength [3]. In contrast, non-linear μFE models are able to directly predict bone strength when an appropriate failure criterion is used [3]. Therefore, non-linear μFE analysis is better suited for estimating bone strength and post-yield behavior [4]. In this study a new software program was developed for non-linear μFE modeling of trabecular bone (TB) strength and failure mechanisms based on a computationally efficient algorithm for linear μFE analysis [5] in conjunction with establishment of a failure criterion for TB tissue. To assess its performance, the serial reproducibility and reliability of TB yield parameters derived from this non-linear μFE approach were evaluated on the basis of *in-vivo* μMR images.

Methods: Yield stress, the stress at which the material deforms irreversibly, can be estimated from the load deformation (stress-strain) curve. Thus in our non-linear μFE model, the first step is to map the stress-strain curve, which is obtained as a best fit to a series of points (the pairs of gradually applied incremental strains and their corresponding stresses). At very small strains, trabecular tissue is assumed to be linear elastic. Therefore a linear model [5] was used to calculate the corresponding stress. However, at larger strains, trabecular tissue exhibits nonlinear stress-strain behavior. Here, a new trabecular bone compressive failure criterion was established on the rationale that trabecular tissue is piecewise linear elastic. The following graduated scale for assigning tissue modulus was applied:

$$\text{Tissue modulus} = \begin{cases} \text{BVF}\times\text{YM}, & \text{if tissue compressive strain} \leq 0.5\times\text{tissue yield strain} \\ \text{BVF}\times\text{YM}\times 80\%, & \text{if } 0.5\times\text{tissue yield strain} < \text{tissue compressive strain} \leq \text{tissue yield strain} \\ \text{BVF}\times\text{YM}\times 30\%, & \text{if tissue yield strain} < \text{tissue compressive strain} \leq 2\times\text{tissue yield strain} \\ \text{BVF}\times\text{YM}\times 5\%, & \text{if tissue compressive strain} > 2\times\text{tissue yield strain} \end{cases}$$

where Young's modulus (YM) = 15GPa and BVF stands for bone volume fraction [5]. For each applied strain, a non-linear system was solved for the corresponding stress using an iterative procedure. At each iteration tissue modulus was adjusted according to the above scheme and the resultant linear system was solved using a recently developed linear μFE program [5]. Boundary conditions were set to represent axial compression with no friction along the transverse directions. Gradually incremental strains were applied in the axial direction to nodes at the top surface. The second step was to fit the stress-strain curve to these points of applied strains and corresponding stresses with cubic polynomials. Subsequently, the apparent yield stresses and strains were obtained based on the 0.2% offset rule [3].

The performance of the non-linear μFE model was evaluated by a reproducibility study conducted on *in-vivo* μMR images of the distal tibia of four healthy volunteers (ages 26-36 years) acquired earlier [7]. The μMR images were obtained at three time points (baseline, follow-ups 1 and 2 over a 3-month period) using a 3D FSE-OSC sequence [6] at 137 x 137 x 410 μm^3 voxel size. All images were first corrected for subject motion and follow-up images were retrospectively registered to baseline images. The resultant images were then manually masked to isolate the TB region and processed to generate grayscale bone volume fraction (BVF) maps as input to the μFE model [7]. Coefficients of variation (CV) and intra-class correlation coefficient (ICC) were calculated as metrics of reproducibility and reliability.

Results and Discussion: The mean common volume retained for μFE analysis after retrospective registration was 61% of the total scanned volume. Cross-sectional μMR images of a volunteer at three scan time points are given in Figs. 1a-f demonstrating good visual reproducibility and anatomical alignment. The μFE models contained 0.47 ± 0.16 million elements requiring 5.36 ± 2.58 hours for analyzing nine strain levels on a desktop computer with four dual processors (i7-2600 3.40 GHz CPUs) and 8 GB of RAM. Examples of simulated strain maps at 0.9% applied strain are given in Fig. 2 showing within-group similarities and between-subject variations. Fig. 3a shows plots of the stress-strain curves obtained from non-linear simulations on μMR images of one subject at three time points. The mean ($\pm$SD) of the axial stiffness, the derived yield stresses and strains were 598.28 ± 135.54 MPa, 3.88 ± 1.05 MPa and 0.84 ± 0.04%, respectively. At the yield point, 1.92 ± 0.14% of the bone tissue volume was strained beyond the tissue yield strain, which is consistent with the Pistoia criterion [8]. CVs averaged over the four volunteers and ICCs of axial stiffness, yield stresses and strains are 3.1%, 1.1%, 2.9%, 99.2%, 98.4%, 99.5%, respectively, suggesting that between-subject variances of axial stiffness and yield stress dominate over their within-subject variances in spite of the narrow age range of the volunteers (Figs. 3b and c).

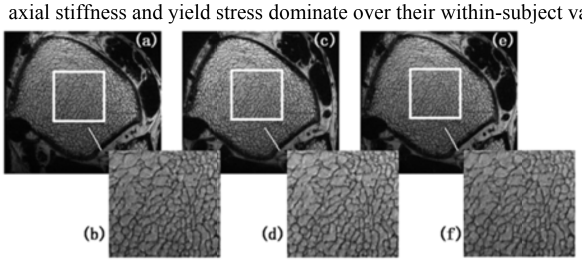

Fig. 1. Cross-sectional μMR images and their magnified sub-regions from a subject at three scan time points: (a,b) baseline; (c,d) follow-up 1; (e,f) follow-up 2, visually illustrate similarities across.

Conclusion: A new non-linear μFE model to predict trabecular bone yield stress and strain was developed and its performance evaluated. The ability to distinguish means between four healthy young subjects suggests that the yield parameters derived from the non-linear model have reproducibility adequate to evaluate treatment effects in interventional studies.

References: [1] Silva et al, J Orthop Res, 1998; [2] Bevill et al, Bone, 2006; [3] Niebur et al, J Biomech, 2000; [4] MacNeil and Boyd, Bone, 2008; [5] Zhang et al, ORS, 2012; [6] Magland et al, MRM, 2010; [7] Bhagat et al, JMRI, 2011; [8] Pistoia et al, Bone, 2002.

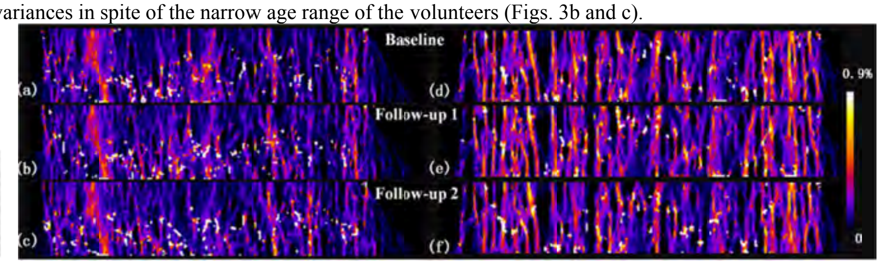

Fig. 2. Simulated strain map projections of a thin slab at 0.9% applied strain for two subjects (a-c) and (d-f) evaluated at three time points: baseline (a,d); follow-ups 1 (b,e) & 2 (c,f).

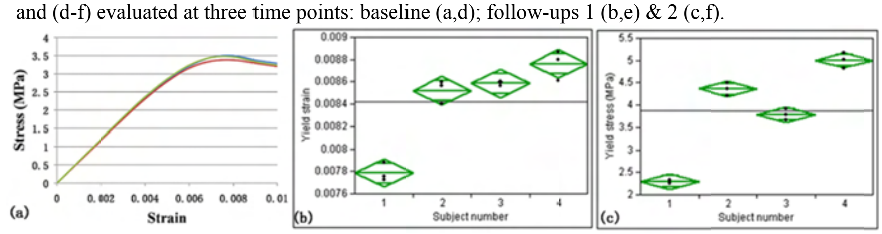

Fig. 3. (a) Plots of stress-strain curves from nonlinear simulations derived from μMR images of a subject evaluated at three time points; (b) Scatter plots of yield strain and (c) yield stress show large variations between subjects and small variations within group, especially for yield stress (ICC=99.5%).

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