Signal-to-Noise Ratio in Spatiotemporally-Encoded (SPEN) MRI employing quadratic phase encoding

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Background The majority of MR imaging techniques is based on encoding and reading the image information in the frequency (***k*-space**) domain. In recent years a conceptually different encoding approach has emerged, based on progressive point-by-point refocusing of the image in the spatial domain using quadratic phase functions [1]. This technique, termed Spatiotemporal-Encoding (**SPEN**), is highly robust against various off-resonance artifacts, such as, ΔB_0 inhomogeneities, multiple chemical sites (i.e., fat & water), or the existence of metallic objects, via its ability to completely refocus all T_2^* relaxations. As a result, SPEN has the potential of extending the reach of MRI into challenging regions such as the human olfactory bulb (see Fig. 1, comparing single-shot Echo-Planar images based on *k*-space encoding vs. SPEN) [2-6]. This work complements an essential part of SPEN by providing a theoretical analysis of its signal-to-noise ratio (**SNR**), and compares it with conventional *k*-space encoding.

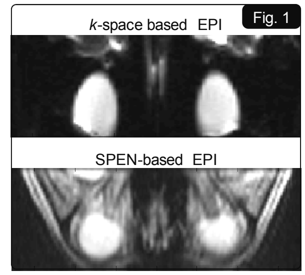

Fig. 1: Human olfactory bulb region imaged using conventional *k*-space encoding vs. SPEN

Theoretical Analysis

SPEN is based on spatial localization of the detected signal using a parabolic phase function which acts to dephase all but a single spatial region located at the parabola vertex. By shifting this vertex across the imaged axis, its structure can be retrieved without the use of Fourier Transformation (**FT**). This strong localization constitutes the basis for much of SPEN's advantages, yet is associated with a lower SNR. A solution is found by relaxing the level of localization, and using wider point-spread-functions (**PSF**) which entail higher signal values. A simple super-resolution (**SR**) reconstruction algorithm can be then employed, to restore the initial image resolution, while re-instating the multiplexing advantage characterizing FT based imaging [7]. The ensuing signal intensity is then proportional to:

$$\textbf{(1)} \qquad Signal^{SPEN} \propto dx \cdot F_{SR}^2 / C_0(F_{SR})$$

where dx denotes the reconstructed pixel size, F_{SR} is the ratio between the width of the initial PSF and dx (hence indicating the level of multiplexing afforded by the SR algorithm), and C_0 is a reconstruction related noise. The experimental / thermal noise in this case, depends on the acquisition bandwidth and on F_{SR}, making the overall SNR,

$$\textbf{(2)} \qquad SNR^{SPEN} = \frac{Signal^{SPEN}}{Noise^{SPEN}} \propto \frac{dx \cdot F_{SR}^2 / C_0}{BW\sqrt{F_{SR}}} \propto \frac{dx \cdot F_{SR}^{2.5}\sqrt{T_a}}{NC_0}$$

where *N* is the number of acquisition readout points. Using identical bandwidth & time parameters, the SNR of *k*-encoded images is given in the following Equation, which, combined with Eq. 2, implies that the SNR of SPEN and *k*- encoding equalizes at $F_{SR}^{2.5} = N$.

$$\textbf{(3)} \qquad SNR^{k-enc} \propto dx\sqrt{T_a}$$

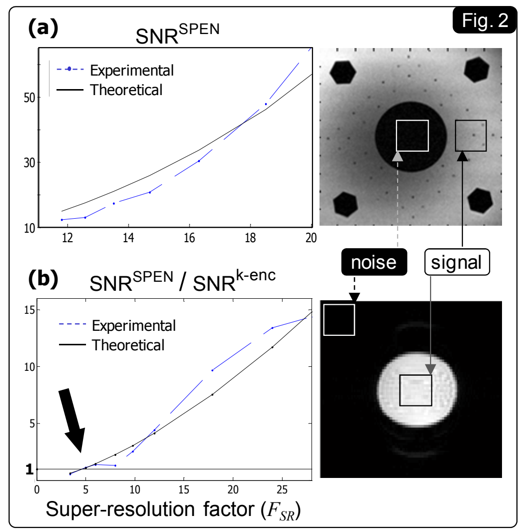

Fig. 2: (**a**) Theoretical & experimental SNR for SPEN. (**b**) SNR ratio between SPEN and *k*-space encoding

Experimental Results

Fig. 2 shows SPEN's theoretical and experimental SNR values as a function of the SR multiplexing level F_{SR} for two water based phantoms. (**a**) SPEN SNR vs. F_{SR} (Eq. 1) [Siemens 7T whole body MRI]. (**b**) SNR ratio between SPEN and *k*-space encoding using $N=70$ along the readout dimension. [single-shot images; Varian vertical 7T MRI]. Good fit is shown in (b) between the experimental result and the theoretical analysis, corroborating that equal (or higher) SNR is available in SPEN as compared to *k*-encoded images when using $F_{SR} \geq 70^{1/2.5} \sim 5.47$ (black thick arrow).

Discussion Previous studies investigating the use of quadratic phase encoding, have all reported the loss of SNR associated with this encoding approach [8-10]. The theoretical analysis hereby presented provides an analytic tool for evaluating SPEN's SNR under various experimental and post-processing conditions, while validating the SNR enhancement that is gained by using the SR reconstruction algorithm presented in [7]. This algorithm opens a new parameter regime, allowing the utilization of SPEN's advantages, while optimizing the ensuing SNR and making its value comparable to that of conventional *k*-space encoding.

References **[1]** Shrot Y and Frydman L, 2005, *JMR*, v. 172, p. 179-190. **[2]** Chamberlain R et al., 2007, *MRM*, v. 58, p. 794-799. **[3]** Ben-Eliezer N, Shrot Y and Frydman L, 2010, *MRI*, v. 28(1), p. 77-86. **[4]** Airaksinen AM et al., 2010, *MRM*, v. 64, p. 1191-1199. **[5]** Goerke U, Garwood M, Ugurbil K, 2011, *NeuroImage*, v. 54, p. 350-360. **[6]** Tal A, Frydman L, 2007, *JMR*, v. 189, p. 46-58. **[7]** Ben-Eliezer N, Irani M, Frydman L, 2010, *MRM*, v. 63, p. 1594-1600. **[8]** Hennig J, Hodapp M, 1993, MAGMA, 1:39:48. **[9]** Pipe James G, 1995, *MRM*, v. 33, p. 24-33. **[10]** Meyerand M E, Wong E C, 1995, *MRM*, v. 34, p. 168-622.

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