Image Denoising Exploiting Sparsity and Low Rank Approximation (DSLR) in Slide Encoding for Metal Artifact Correction

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Introduction: Metal-induced field inhomogeneity is one of the major concerns in magnetic resonance imaging near metallic implants. Slice encoding for metal artifact correction (SEMAC) [1] is an effective way to correct severe metal artifacts by employing additional z-phase encoding steps for each excited slice against metal-induced field inhomogeneity and view angle tilting (VAT). Despite the advantages of metal artifact correction, since noisy resolved pixels are included in image reconstruction, SEMAC suffers from noise amplification. SEMAC with noise reduction [2], which employs a two-step approach (rank-1 approximation along the coil dimension followed by soft thresholding in the slice direction), does not consider noise correlation of coils and results in a direct tradeoff between image accuracy and de-noising. Thus, to further expedite noise reduction in SEMAC, in this work we develop a novel image de-noising algorithm that exploits 1) low-rank approximation using strong correlation of pixels (x-z) in the slice direction (t), 2) Best Linear Unbiased Estimator (BLUE) image combination in the coil direction with noise correlation, and 3) recovery of distorted slice profile using the sparsity of signals in the slice direction with orthogonal matching pursuit (OMP).

Theory: A flow chart of the proposed, image de-noising using sparsity and low-rank approximation (DSLR) is shown in Figure 2. The algorithm consists of the two steps of noise pre-processing (low-rank approximation and BLUE) followed by optimal image reconstruction using the sparsity in the slice direction (OMP).

I. Low-rank approximation: In SEMAC data acquisition (x(or y)z-t), strong correlation of pixels (in x-z) exists in the slice direction (t) for each coil. The pixels in the x-t dimension (colored lines in Fig. 2) are arranged into a single vector: $P_j = [\ell(x_j, t_0), \ldots, \ell(x_j, t_n)]^H$ where x_j is the *j*th pixel in the z direction, and t_n is the nth pixel in the t direction. To exploit the correlation of pixels (in x-z) along the t direction, image pixels in spatio-temporal dimension (xz-t) are rearranged into a single matrix: $L = [P_0, \ldots, P_{m-1}]^H$ where the rows of L correspond to the t direction and the columns of L to the z direction (see Fig. 1a). The zeros in the L matrix are removed and the high signal-to-noise ratio (SNR) pixels are shifted to the center, yielding another permuted matrix L_p (see Fig. 1b). Applying a 1-D Fourier transform along the z direction, Fig. 1c shows that signals are strong in the center. Singular value decomposition (see Fig. 1d) of the L_p followed by the low-rank approximation is performed by minimizing the following constrained optimization problem: $\min\|FL_p - Y\|_2$ s.t. $\mathrm{rank}(L_p) \le r$, where F is a Fourier transformation operator and Y is the measured data. With the nuclear norm, this problem is reduced to $\min\|FL_p - Y\|_2 + \lambda\|L_p\|_*$. **II. Best Linear Unbiased Estimator (BLUE)**: Once noises are pre-processed in the xz-t dimension, SEMAC data is combined in the coil direction using BLUE with noise covariance of coils [4]. BLUE reconstruction is performed using $\tilde{L}_{BLUE} = (C^H \psi^{-1} C)^{-1} C^H \psi^{-1} \cdot \tilde{L}$, where ψ is the noise covariance matrix, C is the coil sensitivity, and $\tilde{L}$ is the solution of the optimization problem in the low-rank approximation. Given the explicit knowledge of noise covariance and coil sensitivity from the pre-scan, BLUE reconstruction enhances noise performance in the coil direction. **III. Orthogonal Matching Pursuit (OMP)**: As SEMAC data undergo the initial two pre-processing steps (low-rank approximation followed by BLUE), images are rarely contaminated by noises and highly sparse in the slice direction. That is, image signals, which result from metal-induced field inhomogeneity, are disseminated to a limited number of resolved data elements. Sparse distorted slice profiles in SEMAC are recovered using compressive sensing algorithm [5] by solving the following constrained optimization problem: $\min\|F_z\tilde{L}_{BLUE}\|_1$ s.t. $\|F\tilde{L}_{BLUE} - Y\|_2 \le \varepsilon$ where $\tilde{L}_{BLUE}$ indicated the noise-suppressed SEMAC images. The l_1 minimization is performed using the OMP algorithm [6]: It starts from an "empty model", builds up a signal model, and picks up an atom at a time, which adds to the signal model the most important new atom. This iterative algorithm is converged until the maximum residual signal in x-f domain reaches the noise level. The flowchart of this OMP algorithm is illustrated in the dash box.

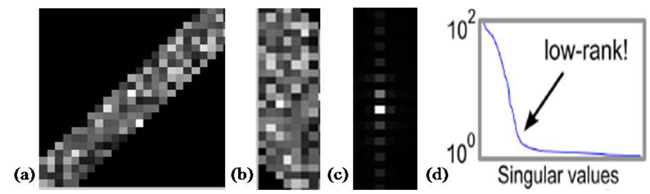

Fig. 1. (a) L matrix (b) L_p matrix (c) x-f space (d) Singular values

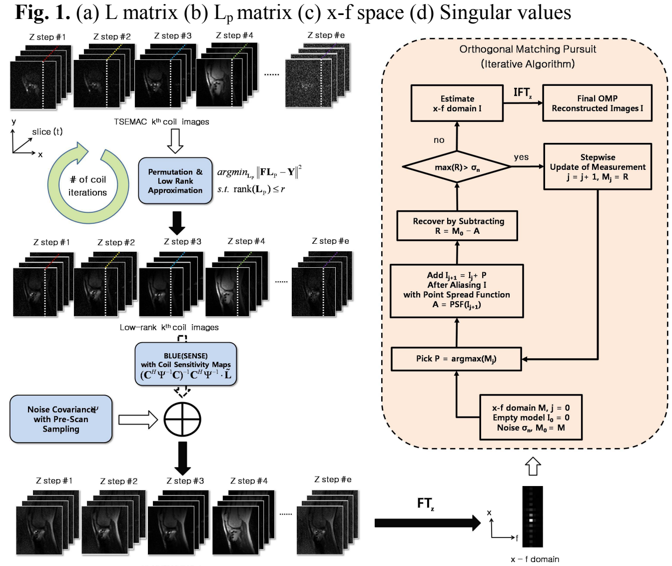

Fig. 2. A flow chart of the proposed DSLR algorithm for SEMAC

Materials and Methods: To investigate the utility of the proposed algorithm, *in vivo* knee data is fully acquired in a volunteer with knee arthroplastics (pedicle screws) using 2D SEMAC imaging at a 1.5 T whole-body MR scanner (Magnetom Avanto, Siemens Medical Solutions, Erlangen, Germany). 640 noise samples are acquired separately before actual imaging data acquisition. The imaging parameters are: matrix, 320x320; FOV, 22 cm; z-phase encoding steps for SEMAC, 8; thickness, 3 mm; # of coil, 8; flip angle, 150°; effective TE, 36 ms; TR, 3500 ms; # of slices, 24; and imaging time, 5.7 min.

Results: Fig. 3 compares three images reconstructed using SEMAC [1], SEMAC with the conventional two-step noise reduction [2], and the proposed DSLR algorithm. The proposed DSLR algorithm outperforms two methods and produces 119% SNR better than SEMAC and 89% SNR better than SEMAC with the conventional two-step noise reduction.

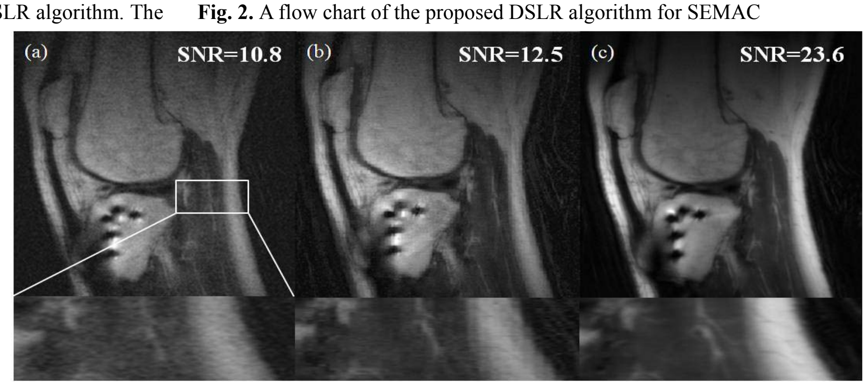

Fig. 3. Comparison of knee images reconstructed using (a) SEMAC, (b) SEMAC with the conventional two-step noise reduction, and (c) with the proposed DSLR algorithm.

Conclusion: We successfully demonstrated that the proposed, novel DSLR algorithm for SEMAC, if compared with conventional de-noising methods, substantially improves SNR and reduces artifacts.

Acknowledgements: Mid-career Researcher Program (2011-0016116), NRF, MEST, Korea and World Class University (WCU) (R31-10008), NRF, MEST, Korea

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