A New Type of Gradient: The Detection Frequency Gradient. A New Capability: Retrospective Shimming

Jonathan C Sharp[1], Scott B King[2], Mike Smith[2], and Boguslaw Tomanek[1]

[1]*Institute for Biodiagnostics (West), National Research Council of Canada, Calgary, Alberta, Canada,* [2]*Institute for Biodiagnostics, National Research Council of Canada, Winnipeg, Manitoba, Canada*

Introduction: Conventional NMR detection results in detected signals having the same frequencies as the precessing spins. However for a time-varying B1 detector field this is not necessarily so, leading to the possibility of *retrospective shimming*, i.e. the detected signal appears shimmed (spectrum has narrow lines) even though the spins themselves are precessing at different frequencies. The dynamic B1 detector field is synthesized retrospectively by a time-dependent weighted combination of NMR signals from a receive coil array. This method exploits array element sensitivity variations within the target volume (as does SURE-SENSE (1)).

Introducing the NMR Detection Frequency: The NMR detector mediates between the RF field of the precessing spins (ω^P) and the detected EMF signal (ω^{EMF}). We will define '*detection frequency*', as: $\omega^D = \omega^{EMF} - \omega^P$. Normally $\omega^D = 0$, however not necessarily, and we will show that a spatial gradient in ω^D is possible.

B$_0$ Field Errors: It is common for the B$_0$ field to have an unwanted ('error') term: $B_0(\boldsymbol{r},t) = B_0^0 + B_0^{EXPT}(\boldsymbol{r},t) + B_0^{ERR}(\boldsymbol{r},t)$, which leads to $\boldsymbol{m}^{CORR} = \boldsymbol{m}^{ACT}\, e^{-i\phi^{ERR}(\boldsymbol{r},t)}$, where $\boldsymbol{m}^{ACT}$ is the actual magnetization, and $\boldsymbol{m}^{CORR} = \boldsymbol{m}^{ACT}\, e^{-i\phi^{ERR}(\boldsymbol{r},t)}$ the desired error-free ('correct') magnetization. The precession errors are given by: $\phi^{ERR}(\boldsymbol{r},t) = -\gamma \int_0^t B_0^{ERR}(\boldsymbol{r},t)dt$.

NMR Detection Process: The NMR detection process is governed by: $emf = -\frac{\partial}{\partial t}\{\boldsymbol{B}_1.\boldsymbol{m}\} = -\frac{\partial}{\partial t}\{|\boldsymbol{B}_1||\boldsymbol{m}|\cos\theta\}$ where normally $\boldsymbol{m}$ is assumed to contain all the time-variation and $\boldsymbol{B}_1$ is static (2,3). Note especially that the $\boldsymbol{B}_1$ detector field does **not** have RF time-variation – if it did an RF EMF would arise from a static $\boldsymbol{m}$!! However, these facts notwithstanding, note the complete symmetry between the two vectors in the equation.

A Modified Detection Process: By a modification of this detection process we can compensate the signal for magnetization precession errors. We write the sample magnetization as: $\boldsymbol{m}^{ACT} = |\boldsymbol{m}^{ACT}|e^{i\theta_m(\boldsymbol{r},t)}$, which includes the precession errors. Now, the precession errors in $\boldsymbol{m}^{ACT}$ would be counteracted, and $\boldsymbol{m}^{CORR}$ recovered, if we had a time-dependent detection field $\boldsymbol{B}_1^{CORR}(\boldsymbol{r},t)$ satisfying this condition:

$$emf^{CORR} = -\frac{\partial}{\partial t}\{\boldsymbol{B}_1^{CORR}(\boldsymbol{r},t)\,.\,\boldsymbol{m}^{ACT}(\boldsymbol{r},t)\} = -\frac{\partial}{\partial t}\{\boldsymbol{B}_1^{STATIC}(\boldsymbol{r}).\boldsymbol{m}^{CORR}(\boldsymbol{r},t)\}$$

This states that $\boldsymbol{B}_1^{CORR}(\boldsymbol{r},t)$, acting as detector for the actual magnetization, would yield the same EMF as a regular static detector field ($\boldsymbol{B}_1^{STATIC}$) detecting the error-free magnetization (the ideal situation). To satisfy this condition it is sufficient that the dot products be equal: $\boldsymbol{B}_1^{CORR}(\boldsymbol{r},t)\,.\,\boldsymbol{m}^{ACT}(\boldsymbol{r},t) = \boldsymbol{B}_1^{STATIC}(\boldsymbol{r})\,.\boldsymbol{m}^{CORR}(\boldsymbol{r},t)$ Omitting the **r**,t dependences for clarity, this becomes: $\boldsymbol{B}_1^{CORR}\,.\,\boldsymbol{m}^{ACT} = \boldsymbol{B}_1^{STATIC}\,.\,\{|\boldsymbol{m}^{ACT}|e^{i\theta_m}\, e^{-i\phi^{ERR}}\}$. Writing $\boldsymbol{B}_1^{STATIC} = |\boldsymbol{B}_1^{STATIC}|\,e^{i\theta_B}$ we can perform the dot product on the right hand side: $\boldsymbol{B}_1^{CORR}\,.\,\boldsymbol{m}^{ACT} = |\boldsymbol{B}_1^{STATIC}||\boldsymbol{m}^{ACT}|\,cos(\theta_B - (\theta_m - \phi^{ERR}))$ and by rearrangement of the phase terms to associate the phase error term ϕ^{ERR} with the **B**$_1$ field (rather than **m**) the solution is:

$$\boldsymbol{B}_1^{CORR}(\boldsymbol{r},t) = \boldsymbol{B}_1^{STATIC}(\boldsymbol{r})\,e^{i\phi^{ERR}(\boldsymbol{r},t)}$$

So our claim is that spin precession errors can be eliminated by a detection field that is the product of any static field and a spatial phase correction term.

Retrospective Shimming using a Detection Frequency Gradient: For the case of a linear gradient inhomogeneity: $B_0^{ERR} = xG_x$ so $\phi^{ERR} = -\gamma x G_x t$, and $\boldsymbol{B}_1^{CORR}(\boldsymbol{r},t) = \boldsymbol{B}_1^{STATIC}(\boldsymbol{r})\,e^{-i\gamma x G_x t}$. From our condition on $\boldsymbol{B}_1^{CORR}$ the error term has been eliminated from the EMF, so the detection frequency is:

$$\omega^D(\boldsymbol{r}) = \omega^{EMF}(\boldsymbol{r}) - \omega^P(\boldsymbol{r}) = \{\omega_0^0 + \omega_0^{EXPT}(\boldsymbol{r})\} - \{\omega_0^0 + \omega_0^{EXPT}(\boldsymbol{r}) + \omega_0^{ERR}(\boldsymbol{r})\} = -\omega_0^{ERR}(\boldsymbol{r}) = \gamma x G_x = xG_x^D$$

which has the form of a linear *detection frequency gradient* $G_x^D = \partial\omega^D/\partial x = \gamma G_x$. In other words, the detected frequency depends upon the spin location within the sample. This is the mechanism for *retrospective shimming*.

Generation of the Time-Varying Detector Field: So given this requirement for a time-varying $\boldsymbol{B}_1^{CORR}(\boldsymbol{r},t)$ detection field, how do we create it? It can be retrospectively synthesized by a time-dependent weighting of a set of EMF signals acquired from multiple detector coils (receive array). This has many similarities with parallel imaging methods such as SMASH (5). An important difference however is that the weightings are not constant, but vary throughout the acquisition window.

Taylor Series Method: For a linear detection frequency gradient we need to create a receive field: $\boldsymbol{B}_1^{CORR} = |\boldsymbol{B}_1^{STATIC}|\,e^{i\theta_B}[cos(xG_x^D t) - isin(xG_x^D t)]$ which can be expanded as a power series in *x*: $\boldsymbol{B}_1^{CORR} = |\boldsymbol{B}_1^{STATIC}|\,e^{i\theta_B}[\{1 - x^2(G_x^D t)^2/2 + \cdots\} - i\{x(G_\omega t) - x^3(G_\omega t)^3/6 + \cdots\}]$, where the powers of *x* can be regarded as the spatial response of the receive fields and the arguments as weighting factors. If we write: $\Phi = xG_x^D t$, then, for a given set of receive field polynomial terms, there will be a value Φ_{MAX} beyond which the approximation breaks down. So for example, when correcting an FID, the maximum available detection frequency gradient decreases linearly with time. **Matrix Method:** This is a more general method. The problem is to find the weights W_C, given the coil field maps A_{PC} and the target field: $B_p = A_{PC}W_C$, where *P* indexes the field point, and *C* indexes the coils. The inverse of A can be calculated: $I_{cp} = \left(A_{cp}^T A_{pc} + \Gamma_{cc}^{\mathrm{T}}\Gamma_{cc}\right)^{-1} A_{cp}^T$, where Γ_{cc} provides regularization to control noise amplification (1,4). The coil weightings are then: $W_c = I_{cp}B_p$. The corrected EMF is obtained from the weighted sum of the acquired EMFs.

Results: Simulation results are shown for $|\Phi| < 135°$ and 4 terms in the Taylor series polynomial. The sample is a single spectral line with T2 decay. **Fig.1:** raw FIDs from the 4 RF Rx coils *(*uniform, *x*, *x*^{2}, *x*^{3}*);* **Fig.2:** FIDs after weighting has been applied; **Figs.3:** un-shimmed FID (i.e. signal from uniform coil), corrected FID (i.e. complex sum of weighted FIDs); the ideally-shimmed FID; **Fig.4:** spectra corresponding to Fig.3.

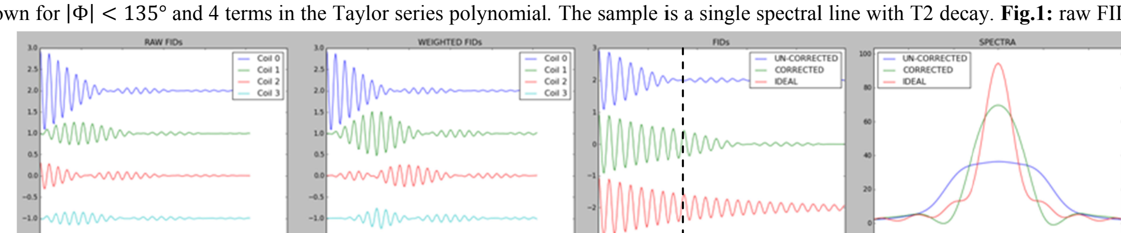

Fig.1 Raw FIDs (4 coils) Fig.2 Weighted FIDs (4 coils) Fig.3 FIDs compared Fig.4 Spectra compared

Discussion: The corrected FID shows complete rephasing (dotted line in Fig.3) where the un-shimmed signal is a null. The correction is less effective at the end of the FID. The corrected line-width shows significant improvement, even though complete rephasing is not achieved. **Conclusions:** A new type of NMR field gradient has been introduced (*detection frequency gradient*). Simulations show that this enables *retrospective shimming*, a unique capability. In principle many types of B0 error (shimming, eddy currents, instabilities) are candidates for correction using this method. Because the method is retrospective each voxel can be corrected individually, which is a unique capability. The method is compatible with all other shimming methods. Implementation requires receive coils with different sensitivity distributions over the target volume (i.e., intra-voxel variations). Design of suitable coil arrays will therefore likely be key in determining range of applicability.

References: 1) Otazo-R NeuroImage 47 (2009) 220-230; **2)** Hoult, D.I. and P.C.Lauterbur (1979) JMR **34**(2): 425-433; **3)** Hoult,D.I. and R. E. Richards (1976). JMR **24**(1): 71-85; **4)** Lin, F. H., K. K. Kwong, et al. (2004). MRM **51**(3): 559-567; **5)** Sodickson, D. K. and W. J. Manning (1997). MRM **38**(4): 591-603.