MMSE optimal non-local motion compensation for compressed sensing cardiac cine imaging using k-t FOCUSS

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Introduction: Compressed sensing (CS) tells us that the perfect reconstruction is possible if the nonzero support in transform domain is sparse and sampling basis are incoherent [1]. By exploiting that dynamic MRI can be sparsified due to the temporal redundancy, we have demonstrated successful application of CS for cardiac imaging [2]. In particular, more accurate prediction using motion estimation/compensation [2] or data-driven optimal temporal sparsifying transforms [3] have proven to be quite effective. However, despite their successes to some extent, there still remain considerable artifacts in edge area when the acceleration factor increases. In this paper, we propose non-local motion compensated k-t FOCUSS algorithm in which more accurate prediction images are provided for k-t FOCUSS than the existing block matching algorithm. The proposed motion compensation is shown optimal in the minimum mean square error sense (MMSE).

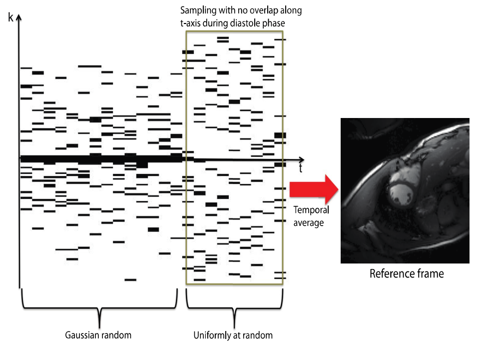

Fig. 1. Proposed sampling pattern

Theory: In cardiac cine imaging, there are significant temporal redundancies along temporal direction. In particular, the spectral support in x-f space is very sparse due to the periodic motion where x denotes the spatial dimension along the phase encoding direction and f denotes the temporal Fourier frequency. With a prediction $\hat{\mathbf{x}}$ of unknown x-f signal $\mathbf{x}$, k-t FOCUSS algorithm [2] can estimate the x-f image as $\mathbf{x}=\hat{\mathbf{x}}+\Delta\mathbf{x}$, where the residual $\Delta\mathbf{x}$ is obtained by iteratively solving the following problem :

$$\min \|\Delta\mathbf{q}_l\|_2, \quad \text{subject to } \|\Delta\upsilon-\mathbf{F}\mathbf{W}_l\Delta\mathbf{q}_l\|_2 \le \varepsilon \tag{1}$$

where $\Delta\mathbf{x}_l=\mathbf{W}_l\Delta\mathbf{q}_l$ and $\mathbf{W}_l$ is a is diagonal weighting matrix at the l-th iteration. Using a Lagrangian multiplier, the solution of (1) can be represented as :

$$\mathbf{x}_{l+1}=\bar{\mathbf{x}}_{l+1}+\mathbf{\Theta}_l\mathbf{F}^H(\mathbf{F}\Theta\mathbf{F}^H+\lambda\mathbf{I})^{-1}(\upsilon-\mathbf{F}\bar{\mathrm{x}}_{l+1}), \tag{2}$$

where $\mathbf{\Theta}_l=\mathrm{W}_l\mathrm{W}_l^H$ and $\bar{\mathbf{x}}_{l+1}$ is a prediction term. More accurate prediction term into k-t FOCUSS promises better quality of reconstruction image because accurate prediction leads to sparser residual. In this paper, accurate motion compensation method called non-local motion compensation is proposed to make the residual signal more sparse and CS reconstruction more accurate. The idea comes from non-local means algorithm in image restoration problem [4]. More specifically, in the proposed non-local motion compensation, a subvector $\mathbf{x}_{I_i}$ at the index set I_i is estimated by solving a weighted least-squares problem. Then the solution is represented as:

$$\hat{\mathbf{x}}_{I_i}=\frac{\sum_{j\in\mathcal{N}_i}\theta(i,j)\mathbf{y}^r_{I_j}}{\sum_{j\in\mathcal{N}_i}\theta(i,j)}=\sum_{j\in\mathcal{N}_i}p(i,j)\mathbf{y}^r_{I_j}, \quad \text{where } \theta(i,j)=\exp(-\frac{\|\mathbf{y}_{I_i}-\mathbf{y}^r_{I_j}\|^2}{2B\lambda^2}) \quad \text{and } p(i,j)=\frac{\theta(i,j)}{\sum_{j\in\mathcal{N}_i}\theta(i,j)}. \tag{3}$$

Here, $\mathcal{N}_i$ denotes the collection of index sets on a reference frame where the similar patches can be found and $\mathbf{y}^r_{I_j}$ denotes the patch from the reference frame at the index set I_i, $B=|I_i|$ and λ is a smoothing parameter. If the reference patches are assumed to have identical prior distribution and measurement $\mathbf{y}_{I_i}$ follows the Gaussian distribution, then we can show that (3) is equivalent to the MMSE estimator. In non-local means algorithm [4], the center pixel of a block is estimated, whereas the entire block is replaced with the estimated blocks in the non-local motion compensation. Moreover, in non-local motion compensation, similar blocks are retrieved from another reference image that usually have high resolution. In our experiment, reference frame was obtained from data itself using diastole phase (Fig. 1). Outside the diastole phase follows the Gaussian random sampling patterns. Since non-local motion compensation outperforms the conventional ME/MC, we incorporate the non-local motion compensation with k-t FOCUSS as a more accurate prediction algorithm. The schematics of non-local motion compensation is shown in Fig. 2. Fig. 2(a) shows dynamic frames along times axis and (b) shows reference frame from diastole phase. In (c), a block in the currently processed frame is estimated in an MMSE sense as a weighted average of similar blocks in reference frame, i.e., $\hat{\mathbf{x}}_{I_i}=\sum_{j\in\mathcal{N}_i}p(i,j)\mathbf{y}^r_{I_j}$.

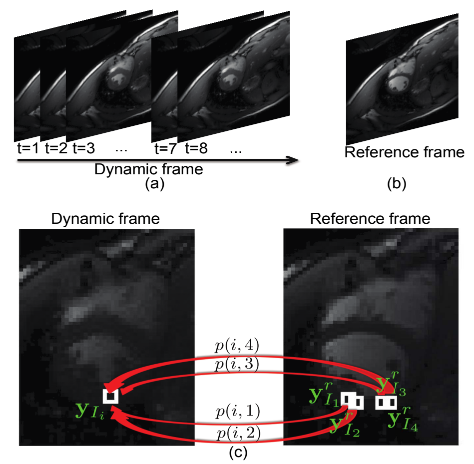

Fig. 2. Schematics of non-local motion compensation.

Result: Fig. 3 shows the reconstruction results at the down-sampling factor of 8. Noticeable improvement can be observed along cardiac wall boundaries in non-local motion compensation compared to the original k-t FOCUSS [3]. Also in x-t slice views, the non-local motion compensation shows the best reconstruction near the cardiac motion.

Conclusion: We propose a non-local motion compensated k-t FOCUSS which generates more accurate prediction images than the existing motion compensated k-t FOCUSS. Non-local motion compensation is shown MMSE optimal and experimental result shows that the proposed algorithm clearly reconstruct the important cardiac structures and improves over k-t FOCUSS.

References: [1] D. L. Donoho, *IEEE Trans. on Information Theory*, vol 52, no 4, pp. 1289-1306, 2006 [2] H. Jung et al. *Magnetic Resonance in Medicine*, vol 61, no 1, pp. 103-116, 2009 [3] H. Jung et al. *Physics in Medicine and Biology*, vol 52, pp. 3201, 2007 [4] A. Buades et al. *in Proceedings of the 2005 IEEE Computer Society Conference Vision and Pattern Recognition*, vol 2, pp. 60-65, 2005

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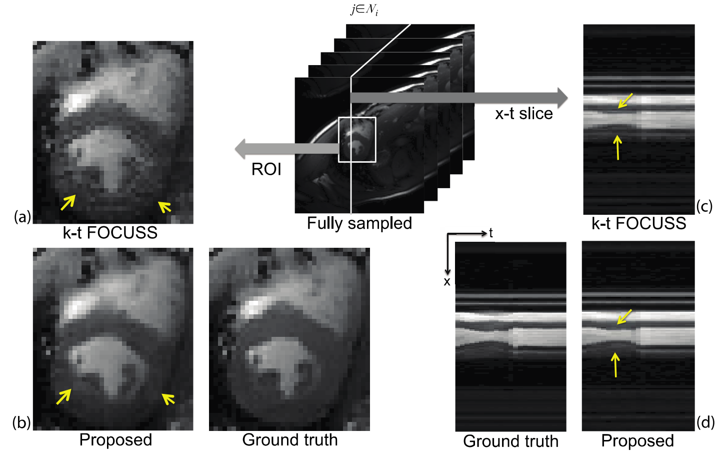

Fig. 3. Reconstruction results at heart region (a)(b) and reconstruction x-t profiles (c)(d).