Effect of truncated sampling on estimated fiber directions in q-space Imaging

B. Wilkins[1], N. Lee[1], and M. Singh[1]
[1]Radiology and Biomedical Engineering, University of Southern California, Los Angeles, California, United States

Introduction: Diffusion Spectrum Imaging (DSI) [1] samples the three-dimensional q-space to yield an ensemble average propagator (EAP) describing the local diffusion of water molecules. To reduce scan time, sampling is frequently limited to a truncated-sphere within a Cartesian grid of q-space. In two-dimensional imaging it has been shown [2] that the corners of k-space contribute to the resolution of an image, provided an interpolation method such as zero-padding is used. The objective of this work was to investigate the consequence of truncated-sphere sampling on estimating fiber directions. Specifically, we simulated the error in resolving a single fiber direction, by systematically changing its orientation, to determine how the corners of q-space impact errors for certain fiber orientations.

Method:
A single compartment Gaussian model mixed with background isotropic diffusion was used to generate noiseless q-space data as proposed in [4], $S(\vec{q}) = S(0)((1-f_0)\exp\vec{q}^T D\vec{q}\tau) + f_0\exp(\vec{q}^T D_o\vec{q}\tau))$. The eigenvalues for a single fiber were set to {1.7, 0.3, 0.3} um^2/sec based on nominal values observed in diffusion measurements of white matter axons and used in [3]. Diffusion time and diffusion gradient duration (Δ/δ = 80ms/35ms) were obtained from [4] and fixed to generate q-space samples within a Cartesian grid. A Rician distribution of noisy diffusion data was obtained with SNR=30 based on the non-diffusion weighted signal. A 7x7x7 Cartesian grid of q-space was selected to result in an optimal truncated sampling with a b_{max} of 4000sec/mm^2 as proposed by [5]. Fully sampled q-space data has a maximum b_{max} of 6800 sec/mm^2 along diagonal directions. A mask limiting the samples within a truncated-sphere was applied to result in DSI 203 data. The EAPs were obtained by a 3D Discrete Inverse Fourier Transform (DIFT). For a truncated dataset, the missing points were zero-filled before applying a 3D DIFT. The full and truncated q-space data were zero-padded to 21x21x21 preserving the conjugate symmetry of the q-space data. To obtain Orientation Distribution Functions (ODFs), 642 radial directions were used. Figure 1 illustrates the simulation geometry; the angles φ and θ define the orientation of a single fiber. Due to spherical symmetry φ and θ were limited to the ranges $0 \le \varphi \le \pi/2$ and $0 \le \theta \le \pi/2$. The angular step between fiber directions (dφ and dθ) was fixed at 1.5°, resulting in 3721 single fiber directions. For both fully-sampled and truncated-sphere cases, the effect of noise (noise-free, or SNR = 30) and zero-padding (none, or 3x q-space sample size) were simulated individually and combined. In total eight different combinations of parameters were simulated (refer to Table 1). The entire simulation was repeated 100 times; the mean of all iterations was taken before further analysis.

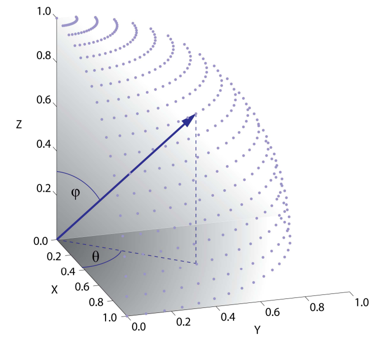
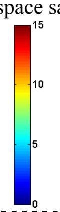
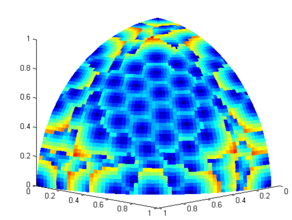
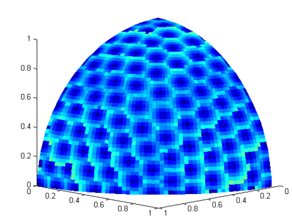
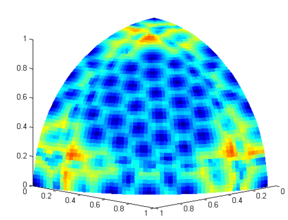
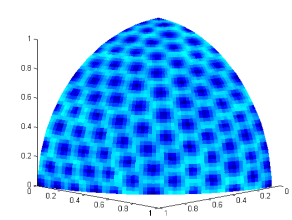
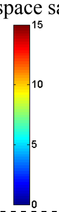
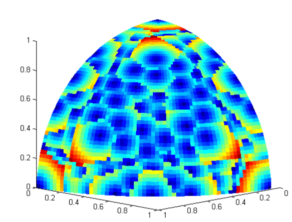
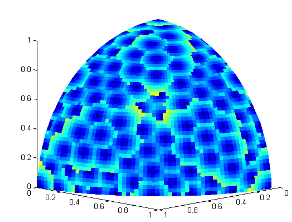
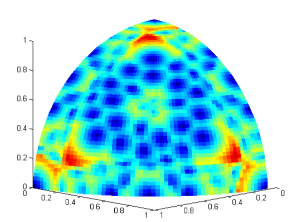
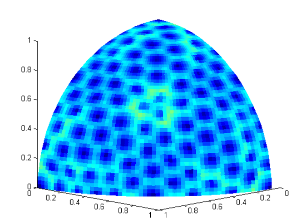


Figure 1 Simulation geometry: dots indicate all fiber directions simulated, one of which is identified by an arrow.

Results: Table 1 illustrates the spatial variation in error when estimating the orientation of a single fiber, for the eight different simulation cases. The magnitude of the error (in degrees) is mapped via the color-bar shown, for all fiber directions indicated in Figure 1. The average error and standard deviation (σ) listed is of all fiber orientations (3712 data points) shown on each corresponding plot.

Table 1	Noise-free data		Noisy data (SNR = 30)	
	No zero-padding (7x7x7)	Zero-padding (21x21x21)	No zero-padding (7x7x7)	Zero-padding (21x21x21)
Fully-Sampled (343 q-space samples) 				
Avg. Error ± σ	4.9° ± 2.6	3.2° ± 1.5	5.2° ± 2.3	3.5° ± 1.3
Truncated-Sphere (203 q-space samples) 				
Avg. Error ± σ	5.3° ± 2.9	3.6° ± 1.8	5.7° ± 2.6	3.9° ± 1.5

Discussion: The results clearly illustrate a fiber-orientation dependent error, which is reduced for full-sampling compared to truncated-sphere sampling, as would be expected. Also, in all cases, zero-padding in q-space is shown to uniformly distribute error that accumulates along orthogonal directions throughout all directions, and reduces the average fiber orientation error compared to non-zero-padding. Lastly, even in the case of noisy data, the corners of q-space, which are removed in truncated-sphere sampling, contribute to reducing fiber orientation error. This is most evident by the increased error in estimating fiber orientation for fibers oriented towards the corners ($\varphi \approx \pi/4$ and $\theta \approx \pi/4$) of the spatial (and thus frequency) domain. The corners of q-space contribute to a spatially uniform pattern of fiber orientation errors, which should be of importance to fiber tractography. Even when zero-padding is used, errors accumulate along the directions of "missing" q-space data in the case of truncated-sampling. Our findings are consistent extensions to the two-dimensional k-space findings reported by Bernstein *et al* in [2]. Note, however that the truncated-sampling scheme would require 40% less data acquisition time than fully-sampled data, and this may justify the slight increase in error that results.

References: [1] Wedeen, V.J., *et al*, Magn Reson Med 54:1377-1386 (2005). [2] Bernstein, M. A., *et al*, J. Magn Reson Imaging 14:270-280 (2001). [3] Tuch, D.S., Magn Reson Med 52:1358-1372 (2004). [4] Yeh, F.-C., *et al*, IEEE Trans Med Imaging 29(9):1626-1635 (2010). [5] Kuo, L.W.,*et al*, Neuroimage 41:7-18 (2008).