Fast Eddy Current Simulation in Thick Split Cylinders of Finite Length Induced by Coils of Arbitrary Geometry

H. Sanchez-Lopez[1], M. Poole[1], L. Liu[1], and S. Crozier[1]
[1]School of Information Technology & electrical Engineering, The University of Queensland, Brisbane, QLD, Australia

Introduction:

In this paper we present a new and fast method to simulate eddy currents in thick and finite length cylinders with a central gap. We assumed that the currents are induced by coils of arbitrary geometry. The method divides thick conducting split cylinders into thin layers (thinner than the skin depth) and expresses the current density on each as a normalized Fourier series. The coupling between each mode with every other is efficiently calculated in Fourier space. In this way, the eddy currents induced in realistic cryostat surfaces by coils of arbitrary geometry can be simulated with a network method. The boundary conditions of the current density at the edges of the domain are appropriately included in such a manner that the continuity equation of the current density is satisfied. The currents induced by a split, actively-shielded *x*-gradient coil were simulated assuming a RF shield and a finite length, split, cylindrical cryostat consisting of three different conducting materials. An accurate simulation is performed in few mins compared to convention methods such as FEM which take considerably longer. The new method presented in this paper can be applied to understand the mechanism of undesired interactions of combined MRI imaging modalities [1] and to explore new gradient coils with minimal cross-talk with hybrid split (or non split) MRI systems.

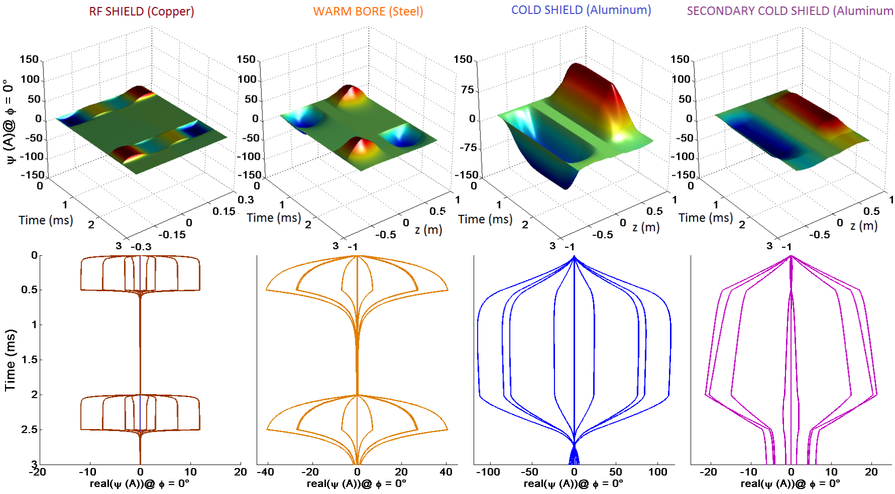
Method: The method is valid for a non-magnetic split (or non split) cylinders made of a linear isotropic conducting media of conductivity σ and immerse in an external magnetic field created by the known source $\mathbf{J}_s(\mathbf{r},t)$. $\mathbf{J}_s(\mathbf{r},t)$ is immersed in the medium of $\sigma=0$ and $\mu_0=4\pi\times10^{-7}$ H/m. The conducting split cylinder is divided into N layers of thickness h, where h is much smaller than the skin depth δ. Each conducting layer into which the cylinder was divided is treated as a thin shell of surface conductivity σh. We assumed that the current that flows in the radial direction is nearly zero ($J_\rho\approx0$), hence no resistive coupling exists between the shells, but that they are inductively coupled. $\mathbf{J}_s(\mathbf{r},t)$ is expressed as a finite Fourier series of orthogonal functions. The boundary conditions and the edges of the domain are enforced to satisfy the continuity equation. The diffusion equation,

$$\mathbf{M}_{\mathrm{ii}}\frac{d\mathbf{s}_{\mathrm{i}}}{dt}+\mathbf{R}_{\mathrm{ii}}\mathbf{s}_{\mathrm{i}}=-\mathbf{M}_{\mathrm{is}}\frac{ds_s(t)}{dt}$$

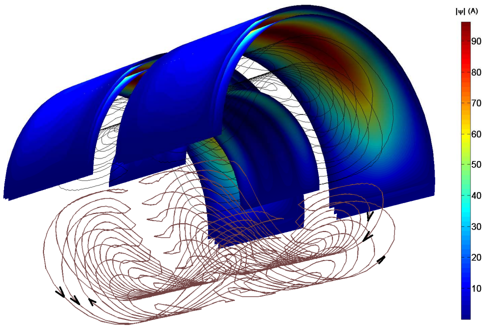
is solved for time-harmonic or transient solution when the coil is driven with an arbitrary current pulse. $\mathbf{M}_{\mathrm{is}}$ is the mutual inductive coupling between the source and the conducting shells and $\mathbf{M}_{\mathrm{ii}}$ and $\mathbf{R}_{\mathrm{ii}}$ are the self inductive and resistive coupling of the conducting shells, respectively [2]. $\mathbf{s}_{\mathrm{i}}$ is the vector that contains the unknown amplitudes of the Fourier modes representing the induced current.

Results and Discussions:

The method was validated against the commercial software FEMLAB using a canonical problem. The canonical problem consisted in predicting the eddy current penetration in a thick and split cylinder induced by double-loop current coils driven by a harmonically varying current. The new method predicted a skin depth of 2.785 mm, that is close to the FEMLAB(δ=2.8 mm) at 1 kHz in a 25 mm thick conducting cylinder of $\sigma=32.26\cdot10^6$ S/m. This result demonstrated the accuracy of the new method when predicting the skin depth. The figure presented below shows the transient and time-harmonic analysis of the eddy currents induced by a split whole-body *x*-gradient coil in a RF shield and cryostat vessels. The *x*-gradient coil produces 10 mT/m and has a central gap of 20 cm. The simulation was completed in 25 min which is comparatively very fast.



Left. Transient analysis of the current induced in each conducting cylinder. *Below.* Split gradient coil and a representation of the absolute value of the current (time-harmonic) induced in each cylinder



We presume that the small $B_{0\text{-shift}}$ is due to the discrete nature of the exciting coil and its inductive coupling with the RF shield. We realize that mostly all the zonal harmonics that might shift and destroy the B_0 spatial profile are originated in the RF-shield. In virtue of the copper RF shield (12μm), this undesired harmonic is dissipated during the ramp-(up/down) time; a larger thickness would produce a longer lasting effect during the flat-top time.

The induced current flows within the conducting domain as a successful consequence of applying the boundary conditions.

The RF shield and the warm bore (WB) produced the largest power. The RF shield and the WB have conductivities values smaller than that of the cold shield (CS) and secondary cold shield (SCS). This indicates that the RF shield and the WB might produce the largest amount of vibration power and therefore might be a significant acoustic source.

Conclusions: A new method to simulate the current induced by coils of arbitrary geometry in thick, split cylinders of finite length has been presented. The inclusion of the boundary conditions in the solution of the diffusion equation fulfils the continuity equation of the current density, thus no current flows outside the conducting domain. The new method is able to accurately predict the skin depth in split cylinders excited by coils of arbitrary geometry. We found that the RF-shield deserves special attention as: a) it is one of the conducting surface close to the ROI; b) we found a strong inductive coupling between the gradient coil and the RF-shield and a weak inductive coupling between the RF-shield and the cryostat, therefore more eddy currents are generated in the RF-shield; c) most of the $B_{0\text{-shift}}$ is generated by the discrete nature of the gradient coil and its inductive interaction with the RF-Shield.

References: [1] Peng, B. J. *et al* Phys. Med. Biol., 55, 265–280, 2010. [2] H Sanchez-Lopez *et al.* in press J. Magn. Reson. , 2010.

Acknowledgements: This work was performed for MedTeQ, a Queensland Smart State funded research centre.