Adaptive compressed MRI sampling based on wavelet encoding

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Introduction: The main idea of Compressed Sensing is to exploit the fact that there is some structure and redundancy in most signals of interest. Clearly, the more we known about the signal and the more the information we encode into the signal processing algorithm, the better performance we can achieve. In this paper, we propose an adaptive compressed MRI sensing scheme that combined wavelet encoding with compressed sensing which originated from the optic image data acquisition [1]. Our approach exploits not only the fact that most of the wavelet coefficients of MR images are small but also the fact that values and locations of the large coefficients have a particular structure [2,3]. Exploiting the structure of the wavelet coefficients of MR images is achieved by replacing the pseudo-random measurements with a direct and fast method of adaptive wavelet coefficient acquisition.

Theory: Given a low-pass scaling function $\emptyset$ and band-pass wavelet function ψ, define the multi-scale atoms: $\phi_{j,k}(t) = 2^{j/2}\phi(2^j t - k)$, $\Psi_{j,k}(t) = 2^{j/2}\Psi(2^j t - k)$,With j and k indexing their scale and position, respectively. For a special choice of $\emptyset$ and ψ, these atoms form an orthonormal basis for L_2, and we have the following multi-scale representation of a one dimensional signal f with length $N=2^M$.$f = \sum_{k=1}^{2^{j0}} u_{j0,k}\phi_{j0,k} + \sum_{j=j0}^{M-1}\sum_{k=1}^{2^j} w_{j,k}\Psi_{j,k}$ with $u_{j0,k} = <f, \psi_{j0,k}>$ and $w_{j,k} = <f, \psi_{j,k}>$,j0≥0, and there are 2^j wavelet atoms per scale j. These wavelet atoms have a multi-scale nesting structure, the support of each $\psi_{j,k}$contains the supports of $\psi_{j+1,k}$and $\psi_{j+1,2k+1}$which induces a binary tree structure on the wavelet coefficients. We say that $w_{j-1,\lfloor k/2\rfloor}$is the parent of $w_{j,k}$; and that $w_{j+1,2k}$and $w_{j+1,2k+1}$ are the children of $w_{j,k}$. Suppose j0=0, which leads a single wavelet tree, and this tree structure can be expressed graphically by the wavelet coefficient tree. Indeed, Wavelet functions can be regarded as multi-scale local discontinuity detectors, and using the nested support property of wavelets at different scales. The Large/small wavelet coefficients indicate the presence of an edge/smooth region in the support of the wavelet. It is straightforward to see that if a coefficient is nonzero or significant then its father and all its ancestors are likely nonzero or significant.

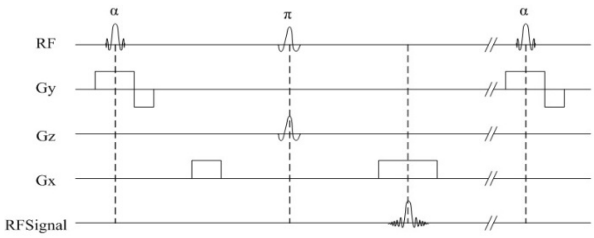

Fig. 1Sequence for 2D single-slice imaging

Methods: For the 2D images, the wavelet-encoded MR data collected under the read gradient is:$S_n(t) = \iint \rho(x,y)\,\varphi_n(y)e^{ixG_x t}dxdy$ [4,5]. Where $\rho(x,y)$is the effective 2D spin density in the slice of interest,$\varphi_n(y)$ is the excited profile in the y direction, it takes scaling function shaped profiles or wavelet function shaped profiles, and $e^{ixG_x t}$ the frequency encoding term in the x direction introduced by the readout gradient. Our adaptive compressed MRI sensing algorithms works as follow, this adaptive sampling process relies on a well-known tree structure of wavelet coefficients, we take advantage of the obtained measurements to decide on which wavelet coefficients should be sampled next.

1. For L-level wavelet encoding along y direction, obtain all 2^{M-L} low resolution measurements by manipulating the RF pulse to generate scaling function shaped profiles, $\{\phi_{M-L,k}(y)\}_{k=1}^{2^{M-L}}$ according to ϕ, and 2^{M-L} the Lth level high resolution data generating by $\{\psi_{M-L,k}(y)\}_{k=1}^{2^{M-L}}$. Ordinarily, L should be bigger if the desired number of measurements N is bigger. Note that the total number of measured data at this stage is 2^{M-L+1}, which is a small fraction of $N=2^M$.
2. From the tree structure of the wavelet coefficients, we know that if father node in the Lth level high resolution has insignificant value, then with very high probability its two children at the next high resolution, the (L-1)th level will also be insignificant. So, Initialize a sampling queue containing the indices of the energy of the measurements in the Lth level high resolution data is bigger than a given threshold [5].
3. Processing the sampling queue until it is exhausted as follows:
   a. According to the position of the sampling queue, index (m,Lth) which at the beginning of the queue, generating wavelet shaped profiles $\{\psi_{M-L+1,k}(y)\}$,k=2m,2m+1 to obtain two samples at the (L-1)th high resolution.
   b. and compute the energy of these two samples, if they are bigger than a given threshold, then add them to the end of the sampling queue.
   c. At the same time, remove the processed index from the queue.

Results: Simulation is carried out to compare the reconstruction result of the proposed adaptive compressed sampling method with the existing 1D variable density random sampling. Figure 2 shows the reconstruction of a brain image with wavelet as the sparse transform when the reduction factor is 2.5602 and 2.4857 respectively, although neither encoding schemes can give the exact reconstruction, the proposed encoding schemes has fewer artifacts and preserves more details than the 1D variable density random sampling.

Conclusion: We have presented an adaptive sampling scheme for MRI data acquisition which can exploits not only the fact that most of the wavelet coefficients of MR images are small but also the fact that values and locations of the large coefficients have a particular structure.The simulation result has shown promising result to accelerate imaging speed with high reconstruction quality.

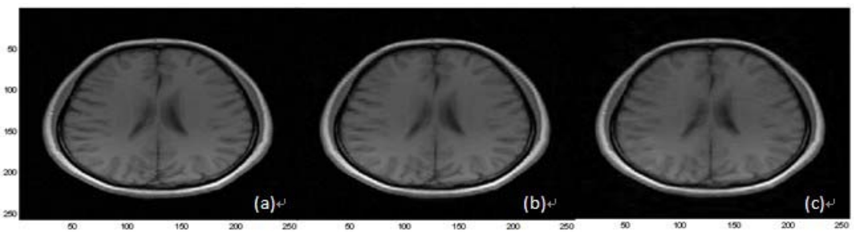

Fig.2 (a) Original brain image. (b) Reconstructed image using the proposed encoding with SNR=25.8825dB.
(c) Reconstructed image using the 1D variable density random sampling with SNR=23.818 dB.

Reference:

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