Monte Carlo analysis of T_1-mixing errors for MSE T_2 mapping

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Introduction and Purpose: Quantitative MRI aims at providing a comparable measure of tissue condition by measuring physical properties instead of image contrast that is influenced by a variety of factors. However, in reality the purpose of qMRI is limited by technical restrictions. In the case of T_2 the measured values are biased from systematic errors depending on the imaging sequence used. Usually T_2 is determined by fitting of mono-exponential decays to multiple single spin-echo (SE) or multi spin-echo (MSE) data. In MSE sequences an echo train is produced after one excitation to speed up the imaging process. In the presence of B_1+ inhomogeneities and non-ideal slice profile (inherent to multi-slice implementations) the echo train strongly deviates from the idealized exponential decay [1, 2]. For phase balanced MSE implementations non-ideal refocusing flip angles (FA) introduce stimulated echoes which alter the signal decay systematically (so called "T_1-mixing"). Nevertheless, some authors fit mono-exponential models to such data, sometimes discarding the first echo which usually deviates most from the relaxation decay [3].
In this work the detailed influences of non-ideal conditions on the measured echo amplitudes of balanced MSE sequences and their impact on estimated T_2 values was investigated by means of computer simulations.

Theory and Methods: The signal magnitudes in a MSE train cannot be expressed analytically in the presence of B_1+ inhomogeneities and non-ideal slice profiles. However, an analytical formula in the z-transform domain does exist [4]. The so-called generating function formalism (GF) is given in eqn. 1.

$$F(z)=\frac{M_0}{2}\left(1+\sqrt{\frac{(1+z\kappa_2)\left[1-z(\kappa_1+\kappa_2)\cos\alpha+z^2\kappa_1\kappa_2\right]}{(-1+z\kappa_2)\left[-1+z(\kappa_1-\kappa_2)\cos\alpha+z^2\kappa_1\kappa_2\right]}}\right) \qquad (1)$$

$$F_{SP}(z)=\frac{1}{Q}\sum_{i=1}^{Q}F(z,\alpha_i)+N \qquad (2)$$

where M_0 is the equilibrium magnetization, $\kappa_1=\exp(-\tau/T_1)$ and $\kappa_2=\exp(-\tau/T_2)$ are the relaxation terms, α is the refocusing flip angle, τ is the inter-echo spacing and T_1 and T_2 are the relaxation times. z denotes a complex variable in the z-domain. Eqn. 2 is an expansion of the formalism to include non-ideal slice profiles (α_i, i=1..Q) and a noise bias. Evaluation of this term for $z=\exp(j\phi)$ ($\phi = 0...2\pi$) and, thereon, applying the FFT yields a discrete time signal corresponding to the echo amplitudes at sampling times $n\tau$. In the Monto-Carlo simulations these ground truth signals were generated for different parameter sets. The slice profile was computed using the SLR algorithm and sampled at 90 points. Subsequently, Rician noise was added and the unknown parameters T_2 and M_0 (in one case also B_1+ and T_1) were estimated by minimizing the residuals (noisy data minus fitted curve) using a L2-norm (Active-set algorithm, Matlab, Natick, USA). This procedure was repeated 10.000 times for each parameter set. The slice profile in the fitting routine was only sampled at 5 time points in order to achieve acceptable fitting speed. Estimation of parameters was done for the mono-exponential model $S=M_0 \exp(-TE/T_2)$ discarding the first echo and the GF approach. In the first experiment the refocusing FA was varied between 90° and 270° assuming ideal slice profile. In the second experiment FA was varied between 90° and 180° assuming Gaussian slice profile. In this case, additionally B_1+ and T_1 were estimated from the data. Further, T_1 was varied between 100 and 3000 ms, T_2 between 20 and 300 ms, and SNR between 20 and 160. The fixed parameters were in all cases τ=10 ms, T_1=1000 ms, T_2=100 ms, α=144°, SNR=80, Gaussian profile. Median values and upper and lower quartiles of all trials were assessed concerning accuracy and precision of the T_2 estimates.

Results: In Fig. 1 the results of the Monte-Carlo simulation are plotted for each parameter set. In (a) the dependence on the actual FA (ideal slice profile) is shown. For deviations from nominal FA up to ± 30° the mono-exponential approach is still quite accurate. For larger deviations only the GF approach yields accurate values. Note that the error is symmetric around 180°. When the slice profile is Gaussian (b), even for ideal FA the mono-exponential estimates deviate strongly from the true values. The GF approach produces accurate and precise results even when, additionally, B_1+ (i.e., the FA) and T_1 are fitted. The estimated FAs match the actual FAs in a wide range but tend to be slightly underestimated in the proximity of 180° (c). T_1 could not be estimated since the signal is in wide range largely independent of T_1. The variation of T_1 (d) shows that for small T_1 values and ratios of T_1/T_2 close to 1 the mono-exponential approach yields accurate values. As T_1 increases, the error approaches approximately 30% (less than 5% for GF). Variation of T_2 reveals a rather constant relative error using the mono-exponential model for physiological T_2 ranges (e) while the GF approach produces accurate values. Despite the computational complexity the noise performance is better for the GF functions approach (f). Similarly, an overestimation of 30% for the mono-exponential model compared to less than 5% for the GF approach indicates the superior accuracy of the GF approach.

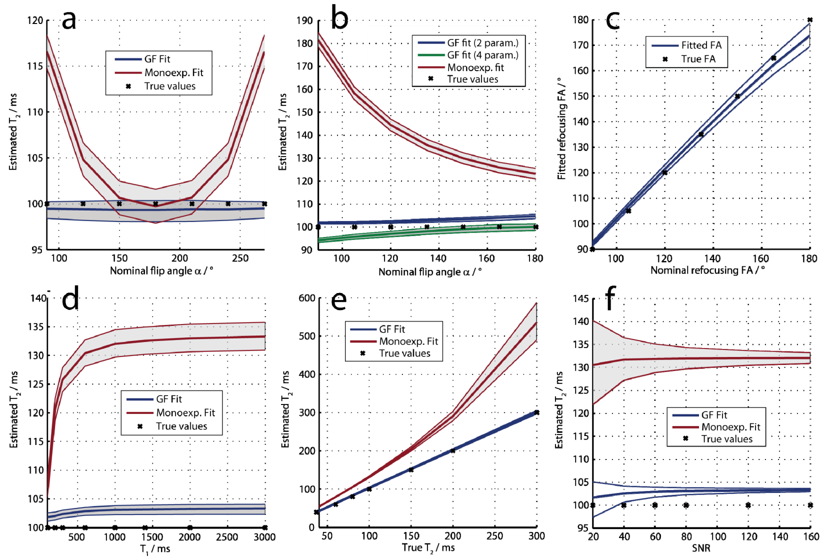

Fig 1.: Estimated median T_2 values for different flip angles and slice profiles (a,b), estimated B_1+ for different angles (c) and T_2 for varying T_1 (d), T_2 (e) and SNR (f). Shaded areas are from lower to upper quartile.

Discussion and Conclusion: Except of the ideal cases of 180° refocusing FA, rectangular slice profile or comparable T_1 and T_2 values, the mono-exponential approach tends to overestimate T_2. The contribution of stimulated echoes (T_1-mixing) slows down the T_2 decay which is, however, a benefit for conventional FSE imaging with long echo trains . The Monte-Carlo simulations reveal that the magnitude of these errors is non-negligible. Care must particularly be taken when T_2 values from different measurements are to be compared as the systematic bias can constitute more than 20% in routine imaging situations. This is true even in the presence of perfectly homogeneous B_1+ field for multi-slice implementations. Furthermore, the results indicate that the slice profile effects dominate over B_1+ inhomogeneities for FA deviations up to ± 30°. The strong dependence of the decay on the actual FA can be exploited as also B_1+ can be estimated for a large range of FAs. Finally, the robustness of the GF fitting algorithm with respect to noise was demonstrated. The proposed GF approach allows to determine the underlying T_2 values by minimizing the adverse effects of the specific experimental conditions.

References: [1] Majumdar S et al. MRM 1987;4:203, [2] Crawley AP et al. MRM 1987;4:34, [3] Mosher TJ et al, Osteoarthr. Cartil 2010;18:358, [4] Lukzen NN et al. JMR 2007;185:71.