The use of Iteratively Reweighted Least Square (IRLS) in the calculation of tissue susceptibility

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Introduction: Quantitative susceptibility mapping can be used to assess brain hemorrhages because the blood breakdown product is strongly paramagnetic. The determination of tissue susceptibility from the measured magnetic field is an ill-conditioned inverse problem, increasing the impact of noise in the field map on the quality of the inversion. In general, the field map noise may be approximated as a Gaussian distribution with a zero mean and a standard deviation inversely proportional to the signal intensity. However, the noise model breaks down in signal void regions associated with hemorrhages, where the phase value, from which the field is derived, is a random variable distributed uniformly between 0 to 2π [1]. In addition, phase unwrapping may further complicate the noise behavior on the field map. In such situations, a manually created mask may be required to exclude voxels with unreliable field estimation [2]. In this abstract, we propose to use the Iteratively Reweighted Least Square (IRLS) algorithm to automatically exclude these bad voxels.

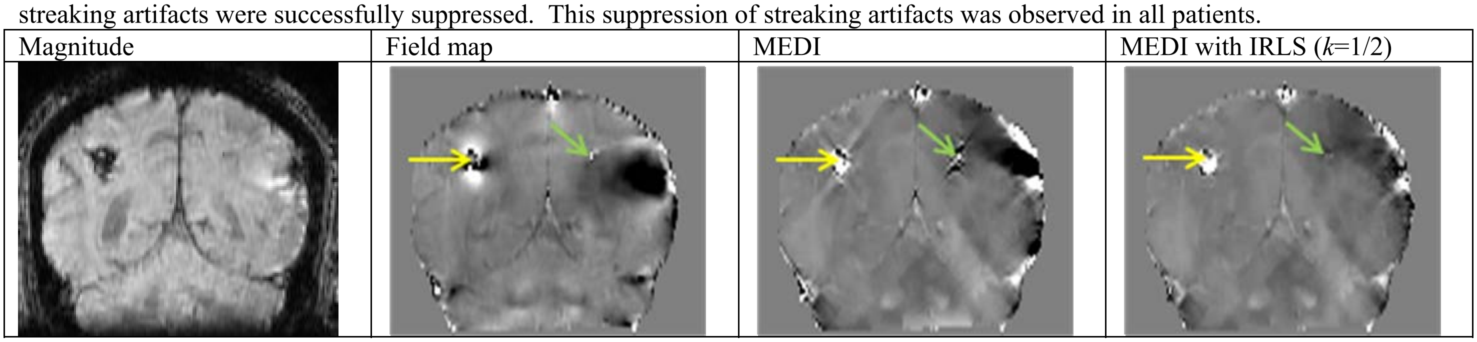
Morphology Enabled Dipole Inversion (MEDI) with IRLS: The original MEDI method minimizes a cost function consisting of a ℓ_2 norm term to ensure data fidelity and a ℓ_1 norm term to sparsify the structural difference between susceptibility distribution and the anatomical image [3]. The IRLS applies this minimization iteratively, in which each iteration has an updated weighting $W^{(n)}$ in the data fidelity:

$$\chi^{*}=\operatorname{argmin}_{\chi}\|W^{(n)}(D\chi-b)\|_2+\lambda|W_G G\chi|_1,$$

where χ denotes susceptibility distribution; b is the measured magnetic field; D is a matrix representing the convolution with unit dipole magnetic field; G is the gradient operator, W_G is a binary weighting derived from the anatomical images and $W^{(n)}$ is the data weighting term. This unconstrained Lagrangian minimization problem is solved using a lagged diffusivity fixed point method, in which χ is updated iteratively:$\chi^{(n+1)}=L(\chi^{(n)})^{-1}\beta$, where $L(\chi)=[(W_G G)^H\cdot 1/|G\chi|\cdot G+2\lambda(W^{(n)}D)^H W^{(n)}D]$ and $b=2\lambda(W^{(n)}D)^H W^{(n)}b$. In addition to $\chi^{(n)}$, $W^{(n)}$ is also updated in each iteration. At the end of the nth iteration, the residual is calculated for all the voxels: $r=W^{(n)}(D\chi\text{-}b)$, and the standard deviation of the residual σ is estimated. For a voxel $\boldsymbol{e}$ whose residual is greater than 6σ, the voxel is considered to be an outlier, and the data weighting is re-calculated as $W(\boldsymbol{e})^{(n+1)}=W(\boldsymbol{e})^{(n)}/r(\boldsymbol{e})^k$, where the exponent k determines the attenuation of the weighting.

Materials and Methods: *Data acquisition*. Five patients with known intracerebral hemorrhage were imaged on a 3T MR scanner using an eight channel birdcage head coil and a multiecho spoiled gradient echo sequence with 4 TEs; uniform TE spacing=5ms and TR=35ms; flip angle=15°; Pixel Bandwidth=400Hz; FOV=24cm; slice thickness=2mm; acquisition matrix size=256x256x64. *Data reconstruction:* The corrected field map was calculated using the Projection onto Dipole Field method [4], and the susceptibility distribution was reconstructed using IRLS with k=0 and k=1/2.

Results: The reformatted coronal views in one patient are presented in Fig. 1. The voxels inside the hemorrhage (yellow arrow) and a few other voxels (green arrow) were corrupted in the corrected field map due to either noise or improper phase unwrapping. These corrupted points further caused severe streaking artifacts on the QSM reconstructed by original MEDI (k=0). With IRLS (k=1/2), the streaking artifacts were successfully suppressed. This suppression of streaking artifacts was observed in all patients.



Discussion: In general, the least square method provides the statistically optimal solution if the noise is Gaussian. However, the field noise is not Gaussian everywhere, and the outlier noise causes streaking artifact in a MEDI susceptibility reconstruction. The proposed IRLS method automatically identifies these outliers, and reduce the data weights for them. In theory, when $0<k<1/2$, IRLS essentially minimizes the data fidelity term using the ℓ_p norm, where $p=2-2k$ [5]. In practice, k can be an arbitrary value greater than 0. If k is set to ∞, IRLS will effectively exclude the outliers. However, when k is greater than 1/2, the corresponding ℓ_p norm minimization problem might not be convex, and the convergence requires further study.

Conclusion: Our preliminary results showed that IRLS is able to suppress streaking artifacts originating from unreliable field measurements, significantly improving the image quality on susceptibility reconstructions.

Ref: [1] Gudbjartsson et al. MRM: 34(6):910-914; [2] Schweser et al. MED PHYS: 37: 5165 [3] Liu et al. ISMRM Proc 2010:4996; [4] Liu et al. ISMRM Proc: 141. [5] Bjorck. Numerical Methods for Least Squares Problems: 4(5):173-175.