

Parallel Imaging Reconstruction II: Non-Cartesian

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Cartesian parallel imaging methods such as SENSE [1] or GRAPPA [2] have been highly successful in accelerating MRI scans, but they can only be applied when the k-space undersampling is regular, i.e. the PSF of the aliased image is a simple comb function. Such regular undersampling leads to well-defined aliasing characteristics in the image domain, and the voxels which overlap can be easily determined. However, when data are acquired along non-Cartesian trajectories, undersampling leads to aliasing artifacts that appear in all directions (the PSF is much more complex), and each voxel in the image domain can potentially alias with all of the other voxels. This is due to the changing degree and direction of the acceleration in an undersampled non-Cartesian trajectory. Thus, the separation of these pixels using non-Cartesian parallel imaging is considerably more complicated than the Cartesian case.

There are many different methods which have been described to perform non-Cartesian parallel imaging. The first method to be introduced was Conjugate Gradient SENSE [3] (CG SENSE). CG SENSE relies on the fact that multi-channel data, combined with information about the coil sensitivities, are redundant, even when the k-space data themselves are undersampled. The relationship between the image and the acquired non-Cartesian k-space data can be written as a matrix equation:

$$\hat{E} \cdot \vec{v} = \vec{m}$$

where \vec{m} is a vector containing the acquired k-space points for each coil, \vec{v} is a vector containing the unknown image voxel values, and \hat{E} represents a combination of coil and gradient encoding. Given that more k-space points are acquired (including the sensitivity encoding of the receiver coils) than number of voxels in the image, it is possible to reconstruct the missing values in the image matrix \vec{v} using the equation above. However, solving this equation directly, i.e. by employing the inverse of the encoding matrix, would require immense amounts memory and computation time due to the large sizes of the matrices and vectors involved. However, the equation can also be solved using the well-known iterative Conjugate Gradient method. One advantage of this approach is that the encoding matrix never has to be written out explicitly; instead, gridding and FTs can be used to solve for the unaliased image. The CG SENSE method is important because it allows one to reconstruct images from arbitrary undersampled trajectories.

Another group of methods introduced for non-Cartesian parallel imaging is the family of non-Cartesian GRAPPAs. Standard Cartesian GRAPPA works by using coil sensitivity variations to generate missing spatial harmonics in the undersampled k-space data. In order for GRAPPA to be applied and calibrated effectively, the sampling in k-space must leave regular patterns of missing datapoints. If this is not the case, a separate GRAPPA weight set is required for each missing point in k-space, a time-consuming and computationally intensive undertaking. More importantly, these GRAPPA weight sets must be calibrated, a challenging task given that each pattern appears exactly once in the dataset, and not multiple times as in standard Cartesian GRAPPA. However, in cases where the non-Cartesian trajectory is highly symmetric, similar patterns do exist, although the patterns are not identical. Non-Cartesian GRAPPA takes advantage of these similar patterns to generate weight sets and reconstruct the missing k-space points. These patterns are geometry dependent and cannot be generalized, and different non-Cartesian GRAPPA methods have been proposed for the radial [4], spiral [5,6], zig-zag [7], and variable density [8] trajectories.

As an example, in radial GRAPPA, segments in the read and projection direction are defined. It is assumed that the geometry within a single segment is Cartesian, and the GRAPPA weight set for this segment is determined using a fully-sampled dataset and applied to the undersampled data. Unfortunately, the reconstruction segment size is difficult to choose such that the reconstructions are accurate; if the segment is too small, the GRAPPA weights cannot be properly determined because there are not enough repetitions of the kernel throughout the segment. However, if the segment is large, the assumption that the data are Cartesian breaks down, and the weights do not reflect the actual radial geometry. Thus, standard radial GRAPPA cannot be used for accurate reconstructions, especially when working with high acceleration factors.

Another option is to use through-time information to supplement the through k-space calibration. This method, known as hybrid through-time/through-k-space non-Cartesian GRAPPA [9], has been demonstrated to yield high acceleration factors because the both geometry of the non-Cartesian k-space and the large number of kernel

occurrences for the calibration are respected. This method can be used with any non-Cartesian trajectory as it does not make assumptions about the geometry of the trajectory. However, this method requires a large number of calibration frames (25-80) as opposed to the standard non-Cartesian GRAPPA, which at most requires a single fully-sampled calibration dataset.

Another method for reconstructing undersampled datasets with non-Cartesian trajectories is PARS [10, 11]. Unlike CG SENSE, PARS is a direct method which reconstructs missing points in k-space, although coil maps are employed in both methods. Instead of generating the missing non-Cartesian points, as in the non-Cartesian GRAPPA methods, PARS directly reconstructs the Cartesian k-space points. In order to accomplish this, source points are chosen from the non-Cartesian data that falls within a specified radius k_r , the so-called local neighborhood, of the "missing" Cartesian point. These source points are combined using weight sets determined from the coil map to reconstruct the Cartesian points.

It is important to note that this list of non-Cartesian parallel imaging methods does not include all of the methods that have been proposed for the reconstruction of undersampled non-Cartesian data. The lack of standard non-Cartesian parallel imaging on most MRI scanners reveals the need for more robust and user-friendly implementations of these techniques (or a new method altogether). Thus, despite the variety of non-Cartesian parallel imaging methods which exist, new ideas which can make these techniques applicable in a clinical setting are still being sought.

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