

# Analysis of Reynolds, Strouhal and Womersley numbers in the healthy thoracic aorta

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**Introduction:** Turbulence and velocity fluctuations of the blood flow are believed to play a role in hemolysis, platelet activation and thrombus formation (2). Previous studies have already assessed turbulence *in-vivo* based on catheter hot-film anemometry, perivascular Doppler ultrasound or MRI (2-5). The Reynolds number has long been used to define critical values for turbulence. Nerem et al. (5) defined *in-vivo* a critical peak Reynolds number proportional to the Womersley number. More recently, those results were amended by a systematic analysis of the influence of Reynolds, Womersley and Strouhal numbers on the development of turbulence *in-vitro* for physiologically realistic pulsatile flow (6). This work investigates the correlation of supra-critical Reynolds number with sex, bodyweight, aortic diameter, pulsatility index and cardiac output based on phase-contrast (PC) MRI in the thoracic aorta.

**Methods:** 2D CINE PC-MRI was performed in 30 healthy subjects (age range = 20-36 years, 9 females) and 8 analysis planes along the aorta (Fig.1, left). The pixel size was 1.24-1.82 x 1.25-1.82 mm<sup>2</sup>, slice thickness: 5 mm, temporal resolution: 24.4 ms. After lumen segmentation and flow quantification based on B-spline interpolation (7), the dimensionless numbers of Reynolds, Womersley and Strouhal were calculated based on the PC-MRI data and the heart rate (Table 1). Following the work of Peacock et al. (6), the critical peak Reynolds number, indicating the transition to turbulence, was derived as:

$$Re_{peak}^c = 169\alpha^{0.83} St^{-0.27}$$

The subject specific supra-critical Reynolds number ( $Re_{peak} - Re_{peak}^c$ ) was calculated and statistically correlated with sex, bodyweight, aortic diameter, pulsatility index and cardiac output.

**Results:** The spatial analysis of supra-critical Reynolds number (Fig.1) revealed aortic locations with mixed distributions (ascending aorta), negative distributions (aortic arch, proximal descending aorta) and positive distributions (descending aorta). Statistical analysis of the supra-critical Reynolds number demonstrated a significant difference between male and female subjects (Fig.2). Further analysis (Table 2) revealed that bodyweight, aortic diameter, pulsatility index and cardiac output were significantly different between the male and female subjects and presented a significant correlation with the supra-critical Reynolds numbers (except for pulsatility index). Regression analysis (Fig.3) showed a positive correlation between bodyweight and cardiac output, and the supra-critical Reynolds number.

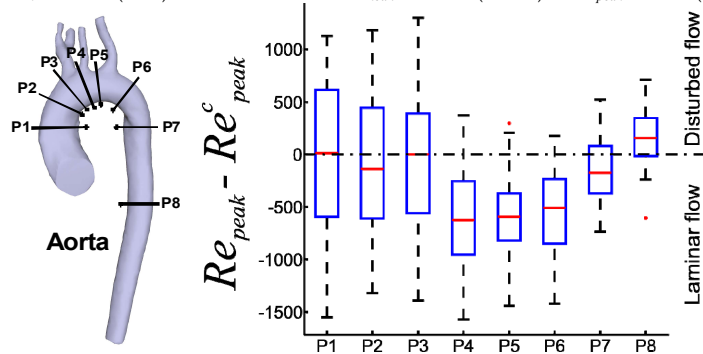
**Discussion:** This study has limitations due to the definition of critical Reynolds numbers based on previous studies using *in vitro* modeling (6). The *in-vivo* situation (e.g. vessel compliance, rheology) may not allow to consider the supra-critical Reynolds number as a strict indicator of the presence of turbulence. Comparative studies with alternative techniques would be needed to assess the sensibility and specificity of the supra-critical Reynolds number to disturbed flow patterns. Nevertheless, the spatial analysis revealed that the descending aorta might be more prone to disturbed flow patterns than the aortic arch. Furthermore, correlation analysis seems to indicate that male subjects of this study are more likely to present onset of turbulence. This disparity could be further tracked down to the subjects' bodyweight, aortic diameter and cardiac output that were all significantly different between male/female and significantly correlated with the supra-critical Reynolds number. These findings indicate that these factors might increase the risk for the presence of disturbed flow patterns. Larger and more diverse populations are needed to confirm these results and identify alterations due to pathologies.

**References:** 1.Gnasso A, et al. Circulation 1996;94, 2.Stein Pd, et al. Circ Res 1974;35, 3.Dyverfeldt P, et al. J Magn Reson Imaging 2008;28, 4.Elkins CJ, et al. Int J Heat and Fluid Flow 2004;25, 5.Nerem RM, et al. Cardiovasc Res 1972;6, 6.Peacock J, et al. Exp Fluids 1998;24, 7.Stalder Af, et al. Magn Reson Med 2008;60

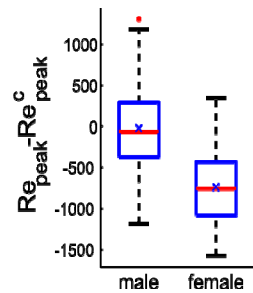
**Table 1: Dimensionless numbers for pulsatile flow**

Reynolds number	Womersley number	Strouhal number
inertial forces / viscous forces	unsteady forces / viscous forces	dimensionless stroke volume
$Re(t) = \frac{\rho v(t) D(t)}{\mu}$	$\alpha = \frac{D_m}{2} \sqrt{\frac{\rho \cdot 2\pi f}{\mu}}$	$St = \frac{D_m}{2} \frac{f}{V_p - V_m}$

with: Density:  $\rho = 1055 \text{ kg/m}^3$ ; Dyn. viscosity:  $\mu = 4.5 \cdot 10^{-3} \text{ cp}$  (1); Mean axial velocity:  $v(t)$ ; Diameter:  $D(t)$ ;  $D_m = \text{mean}(D(t))$ ;  $v_p = \text{max}(v(t))$ ;  $v_m = \text{mean}(v(t))$ ; Heart rate:  $f$ ,  $Re_{mean} = \text{mean}(Re(t))$ ;  $Re_{peak} = \text{max}(Re(t))$



**Fig.1:** Boxplot of the subject-specific difference between measured peak Reynolds number and calculated critical peak Reynolds number. Positive values correspond to supra-critical Re numbers and indicate onset of turbulence. The median (red line), lower/upper quartiles (blue box), most extreme values within 1.5 interquartile range (dashed vertical lines) and outliers (red points) are shown for the 30 subjects.



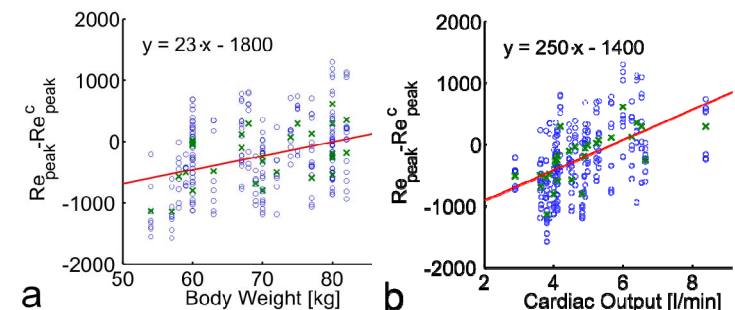
**Fig.2:** Boxplot of the supra-critical peak Reynolds number for male (n = 21) and female (n = 9) subjects.

**Table 2:**

	All / Male / Female	Correlation with $Re_{peak} - Re_{peak}^c$
Bodyweight [kg]	69.7 / 72.4 / 63.3 *	r = 0.34 **
Aortic diameter [mm]	21.9 / 22.5 / 20.7 *	r = 0.48 **
Pulsatility Index	4.1 / 4.3 / 3.6 *	r = 0.29
Cardiac Output [l/min]	4.7 / 5.2 / 3.7 *	r = 0.53 **

\* significant male/female difference (p<0.05)

\*\* significant correlation (p<0.05)



**Fig. 3:** Correlation of supra-critical peak Reynolds number with bodyweight and cardiac output. Circles indicate data for all subjects and analysis planes. Crosses indicate data for all subjects but averaged over the analysis planes.