On the Quality Evaluation for Images reconstructed by Compressed Sensing

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Introduction:

Due to the non-linear and non-stationary nature of compressed sensing algorithms ([1,2]) well known image quality parameters like resolution cannot be determined in a conventional way. Recently, a method was proposed to determine local Point Spread Functions (PSF) by a linear approximation of the non-linear algorithms ([3]). However, the validity of the linear approximation has not been investigated. In this work, we propose a robust technique to determine whether a local approximation of the compressed sensing reconstruction exits linearity and hence whether the concept of a local linear approximation can be applied.

Theory:

Compressed Sensing reconstructions result in an individual PSF for each pixel of the discrete image grid because of the non-stationary nature of the algorithms ([3]). The non-linear nature of the reconstruction is depicted in Fig. 1 which shows a simulation where the real part of one reconstructed pixel is shown as a function of real and imaginary parts of the original signal value in that pixel (CS-algorithm was similar to [4]). If the compressed sensing algorithm can be described by a differentiable function $\bar{f}(\bar{K}) = \bar{B}$, ($\bar{K}=\{K_1,...,K_D\}$ = arbitrary set of k-space values, $\bar{B}=\{B_1,...,B_N\}$ = reconstructed pixels in image space, *N>D*), $\bar{f}(\bar{K})$ can be approximated by an expansion around the actual measured set of k-space values $\bar{\xi}$:

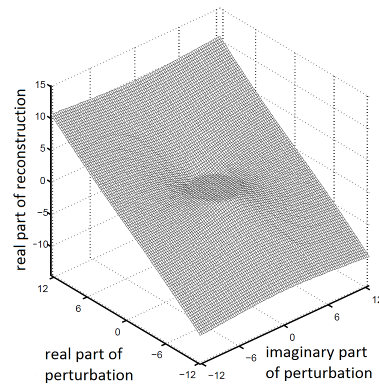

Figure 1: Visualization of functional behavior for one pixel.

$$\bar{f}_m(\bar{K},\bar{\xi}) = \sum_{k=0}^{m} \frac{1}{k!}\left[(\bar{K}-\bar{\xi})^T \nabla\right]^k \bar{f}\Big|_{\bar{\xi}}, \quad \left[(\bar{K}-\bar{\xi})^T \nabla\right]^k = \left[\sum_{i=1}^{D}(K_i-\xi_i)\partial K_i\right]^k$$

If $\bar{\xi}$ is changed only by a sufficient small perturbation $(\bar{K}-\bar{\xi})$ the expansion can be considered as linear and the 1st order approximation represents a satisfactory description for $\bar{f}$ at $\bar{K}$.

Method:

The theory above was used to calculate a PSF for each reconstructed pixel by applying a small local perturbation (Fig. 2, [3]). As can be easily realized, the technique requires the existence of a valid linear approximation. In order to determine whether this condition is fulfilled, the PSF-estimation simply has to be repeated for a doubled (half)-sized perturbation. The perturbation is small enough if the second PSF has exactly twice (half) the magnitude of the initially calculated PSF for every pixel.

An estimation of the PSF and the corresponding linearity-validation was exemplarily performed using a cine dataset (Siemens Magnetom Trio, Erlangen, Germany; TrueFisp, TE/TR=1.4/3.1ms, 126x168, FOV 340x340 mm^2, α=50°). Sparse difference images were calculated by subtracting a single timeframe image from an average image for all timeframes. k-space values were randomly undersampled (3x) in phase encoding direction and the validation test was performed using a compressed sensing algorithm for l_1-minimization similar to [4] .

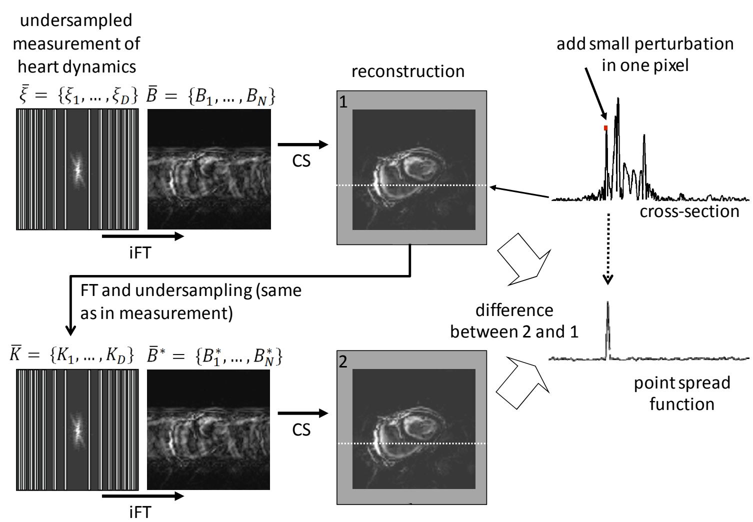

Figure 2: Determination of PSF approximation: The result of a CS-reconstruction (e.g. sparse image of the dynamics of a heart beat) is perturbated by sufficient small amplitude in one single pixel. The image is transformed back to k-space and the equivalent subset of k-space-values (same undersampling scheme like for the measurement) is pulled up for a second CS-reconstruction. Result 1 is subtracted from result 2 which leads to a (linear) approximation for the PSF in that pixel. The method is repeated for every pixel of the image.

Results:

Figure 3 shows exemplarily the calculated PSF for a perturbation of 0.1% of the initial complex value in one pixel (PSF1, top row) and the corresponding PSF for a doubled-sized perturbation (PSF2, bottom row). As expected, the PSF is exactly doubled by using twice the perturbation in each of the reconstructed pixels of the phase encoding direction. The difference between PSF1 and PSF2 is smaller than the numerical precision of the simulation. For all pixels in our data set, a perturbation could be found small enough to fulfill the linear approximation which justifies the PSF approximation in this exemplary case.

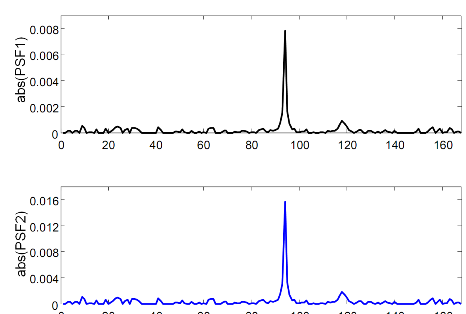

Figure 3: Test of linearity: The simulation did not leave the linear area.

Conclusion:

By means of the proposed PSF approximation it is possible to evaluate the image quality of the results of non-linear, non-stationary reconstruction algorithms such as Compressed Sensing. Varying the perturbation determines whether the linear approximation is valid and whether a differentiable functional behavior is given for the compressed sensing reconstruction. The determination of a PSF is of significant importance for Compressed Sensing reconstructions, as any loss of resolution in comparison to an ordinary Nyquist-sampled measurement can be identified ([3]).

References:

[1] Donoho, IEEE Trans. Inform.Theory 52: 1289-1306 (2006); [2] Lustig, Magn. Reson. Med 58: 1182-1195 (2007); [3] Wech, Proc. Intl. Soc. Mag. Reson. Med. 18: 4885 (2010); [4] Ma, IEEE Computer Vision and Pattern Recognition: 1-8 (2008)