Fourier Series Network Method for 3D Simulations of Eddy Currents Induced in Multilayer Cryostats by Arbitrary Coils

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Introduction In this work we present a new method for eddy current simulation and apply it to a system that is currently under construction. It is based on the network method for eddy current simulation [1] which splits thick cylinders into several thin cylindrical surfaces and calculates the mutual interaction between layers to accurately model the skin effect. The basis functions used here were a Fourier series in the z and φ directions which allowed us to simulate the eddy currents produced by any coil [2]. To demonstrate this, results for an X gradient coil are presented. We simulate the multiple layers of the cryostat structure of a novel high-temperature superconductor (HTS) magnet system for head-only imaging [3]. The magnet wires will be wound from bismuth strontium calcium copper oxide (BSCCO-2223) tape and will be conductively cooled without cryogens to an operating temperature of 20K. The results of these simulations will provide useful information about the interaction between the switching gradient coils and the HTS magnet.

Methods The electromagnetic problem is shown in Fig 1. An asymmetric, actively-shielded, torque-balanced, cylindrical gradient coil was designed with minimal power on the surfaces shown in Fig. 1 a). The wire paths were generated and used for the eddy current simulations. The finite-length thick cylinders were split into multiple thin sub-layers (see Fig 1 c)) such that the layer thickness was less than 1/5th of the characteristic skin depths at 1 kHz. The eddy currents flowing on each sub-layer was expressed as a weighted sum of basis functions that have a sinusoidal variation in z and φ. These basis functions are essentially the same as those used by Carlson to design gradient coils [4] and are inherently of finite length in the z-direction. In this way, the self-inductances ($\mathbf{M}_{ii'}$, $i = i'$), mutual-inductances ($\mathbf{M}_{ii'}$, $i \neq i'$) and resistances ($\mathbf{R}_{ii'}$, $i = i'$) of every basis function with every other is accurately and efficiently calculated in Fourier space, avoiding the singularities that otherwise occur on double integration in real space from 1/|**r**-**r**'|. Also, the mutual-inductances between the coil wire elements and each basis function ($\mathbf{M}_{is}$) were calculated.

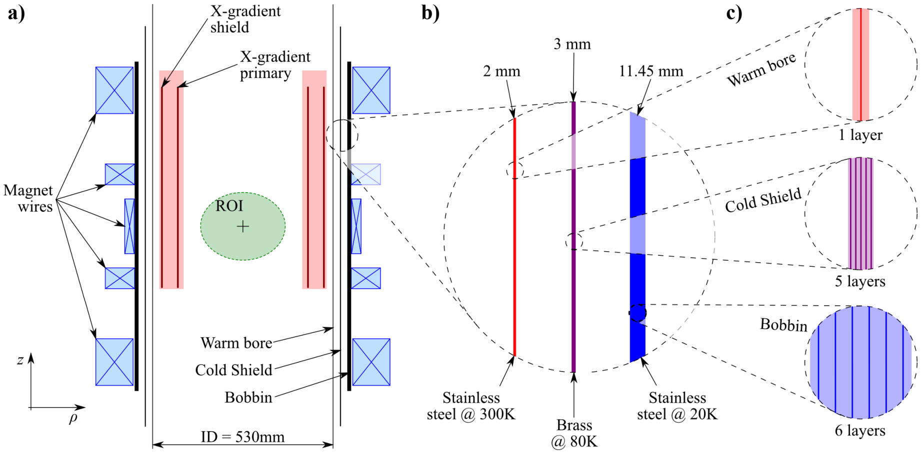

Figure 1. a) Cross-sectional scale drawing of the HTS magnet and gradient coil structures. The region of interest (ROI), gradient coil unit, X-gradient primary and shield layer, warm bore, cold shield, bobbin and magnet wires are shown. b) Zoomed cross-section of the cryostat structure and c) the splitting of the thick cylinders into multiple thin sub-layers.

$$\mathbf{M}_{is}\frac{d\mathbf{s}_s(t)}{dt} + \mathbf{M}_{ii}\frac{d\mathbf{s}_i(t)}{dt}\mathbf{R}_{ii}\mathbf{s}_i(t) = 0 \qquad (1)$$

$$\mathbf{s}_i(\omega) = -i\omega\left(i\omega\mathbf{M}_{ii} + \mathbf{R}_{ii}\right)^{-1}\mathbf{M}_{is}\mathbf{s}_s \qquad (2)$$

28 basis functions (14 each of sine and cosine) were used in the z-direction, which provided good convergence, resulting in a total of 336 basis functions. The eddy currents were then simulated at ω = 1 kHz excitation frequency (transient analysis is also possible [2]) by inversion of the diffusion equation (1) using Equation (2).

Results The outputs from of Equation (2) are the weighting factors for the basis functions of the eddy currents, $\mathbf{s}_i$. The vector field of eddy currents can then be reconstructed from these weights which are difficult to represent graphically. Instead we show the variation of current density with axial, z, and radial, ρ, positions separately, with known cos φ variation. These data are shown in Figure 2 a) and b) respectively. Additionally, the total amount of power deposited in each of the three layers of the cryostat structure were simulated to be 6.1, 0.98 and 0.01 W for the warm bore, cold shield and bobbin respectively.

Discussion and Conclusions This method allows the simulation of the eddy currents in multiple thick cylinders due to arbitrary coils, where previous methods have been restricted to just zonal coils made of concentric circular loops. Standard finite element analysis methods were considered infeasible for modelling such a system because an extremely large number of mesh elements would be required. This method reproduces the skin effect and passive magnetic shielding through the radial direction and has been validated [2]. With this 3D eddy current information the eddy current induced magnetic field can be accurately predicted as well as the heating and potentially the vibrational forces that occur in each layer of magnet cryostat structures. This information will be used in the design and manufacture of a HTS head-only MRI system.

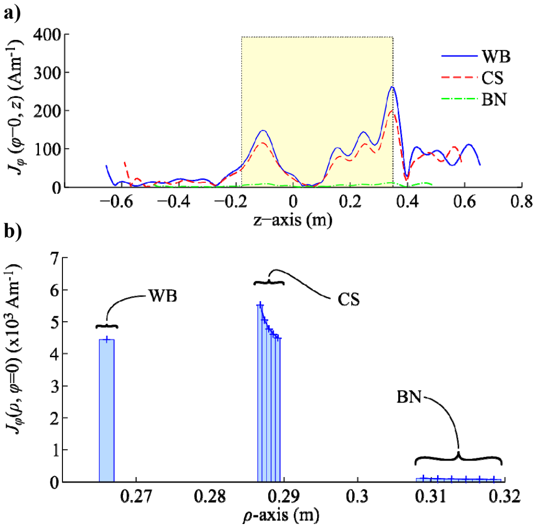

Figure 2. Eddy currents in φ-direction (at φ=0 with cos φ variation) as a function of a) z-position and b) ρ-position for each layer of the cryostat structure. Shaded box represents the axial extents of the gradient coil.

References [1] Takahashi, T. *IEEE T Magnetics*, (1990) **26**, 893-896 [2] Sanchez Lopez, H. et al. *J Magn Reson*, (2010) in press [3] Kitaguchi, H. et al. *IEEE T Appl Supercon,* (2010) **20**, 710-713 [4] Carlson, J. W. et al. *Magnet Reson Med*, (1992) **26**, 191-206.

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