Location Constrained Approximate Message Passing (LCAMP) Algorithm for Compressed Sensing

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Introduction: Fast iterative thresholding methods [1,2] have been extensively studied as alternatives to convex optimization for high-dimensional large-sized problems in compressed sensing (CS) [3]. A common large-sized problem is dynamic contrast enhanced (DCE) MRI, where the dynamic measurements possess data redundancies that can be used to estimate non-zero signal locations. In this work, we present a novel iterative thresholding method called LCAMP (Location Constrained Approximate Message Passing) that combines a non-zero location assumption and an approximate message passing term to previous methods [4]. The method can reduce computational complexity and improve reconstruction accuracy. We demonstrate the proposed reconstruction using 4D breast DCE MRI data, of a size that is often challenging for constrained reconstructions.

Methods: In DCE MRI a set of T_1-weighted images is acquired, starting before a contrast agent injection and continuing over the tissue contrast uptake. When measurements are averaged over time, the signal in a pixel is also averaged and therefore becomes different from the true signal dynamics. The time averaged image, however, can depict the location of non-zero signals in both image and wavelet domains. We utilize this non-zero location (support region) assumption to make the CS reconstruction more effective.

LCAMP Algorithm

$r = N/n$; $\quad c = supp(M)/N$; $\quad$ { Initialization }
$m^k = m^0$; $\quad z^{k-1} = y - \Phi\Psi^T m^0$;

While halting criterion *false*

$\quad z^k \leftarrow y - \Phi\Psi^T m^k + r \cdot c \cdot z^{k-1}$; $\quad$ { Update Residual }

$\quad m^{k+1} \leftarrow (m^k + \Psi\Phi^* z^k) \cdot M$; $\quad$ { Apply Non-zero Mask }

$\quad k \leftarrow k + 1$;

end while

$m \leftarrow m^k$;

Fig 1: *LCAMP (Location Constrained Approximate Message Passing) algorithm.*

Fig. 1 describes the LCAMP algorithm. y is measured data, Φ is the undersampled Fourier transform (n × N), Ψ is the wavelet transform (N × N), and M is the non-zero location mask (1 for non-zero and 0 for zero). supp(M) is the number of non-zeros in the mask M. Note that the approximate message passing term ($r \cdot c \cdot z^{k-1}$) is added when updating the residual [4], and the non-zero location mask is applied when updating m^k. The method typically converges after 2 to 6 iterations, and each iteration only requires two Fourier-wavelet transform operations making the whole reconstruction extremely fast. The typical reconstruction time for a single image (256 × 96) was 250 msec using MATLAB. No regularization parameters are necessary. The residual ($y - \Phi\Psi^T m^k$) becomes worse when the non-zero location mask is sparser and/or the measurement is noisier. Here, we monitor the residual at each iteration, and stop the reconstruction when the difference in residuals is not considerable.

Breast DCE data (magnitude and multi-coil combined) were retrospectively undersampled. The matrix size was 256×256×96×20 ($N_x \times N_y \times N_z \times t$) with a temporal resolution of 11 sec. Fig. 2 shows a k-space sampling density and actual sampling patterns. The sampling density has four different regions: a fully sampled region and three randomly sampled regions with sampling factors (R = 4, 8, and 12). Each region becomes fully sampled when averaged over R frames (e.g. the smallest region becomes fully sampled when averaged over four frames), and therefore the fully sampled data consist of regions with different time averages. The fully sampled data were then used for the non-zero location mask and the initial estimation. Adaptive wavelet thresholding used in image denoising [5] was applied to compute the non-zero location mask. We have developed a plug-in (DCE Tool) for OsiriX and the plug-in was used to generate semi-quantitative maps such as initial slope and area under the curve (AUC). The initial slope was computed using 3 time frames (6th, 7th, and 8th) and the AUC was computed using 15 time frames (from 6th to 20th).

Results and Discussion: Fig. 3 shows a representative slice reconstructed by LCAMP. The net acceleration factor (R) is 11 and this is able to make a much higher temporal resolution image (1 sec) than original (11 sec). Magnitude, initial slope map, AUC map, and time-intensity curve are shown to compare original and LCAMP. The identical window levels were applied for both original and LCAMP. The reconstruction not only maintained good image quality but also kept almost identical signal dynamics even in a highly undersampled situation (R=11). The time-intensity curve was computed from the average signal over a tumor ROI (indicated by arrows).

There are many other applications that can use the non-zero location assumption. T_1 and T_2 relaxation mapping could be such an example as well as functional MRI applications. We believe the same reconstruction can benefit those applications.

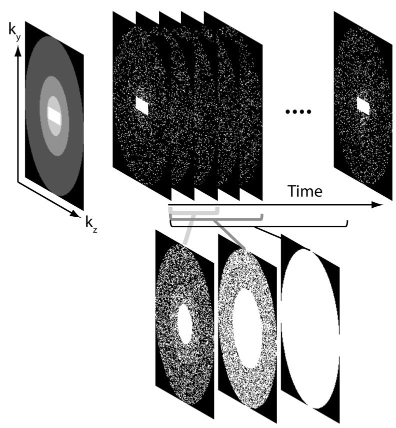

Fig 2: *ky-kz-t sampling pattern.*

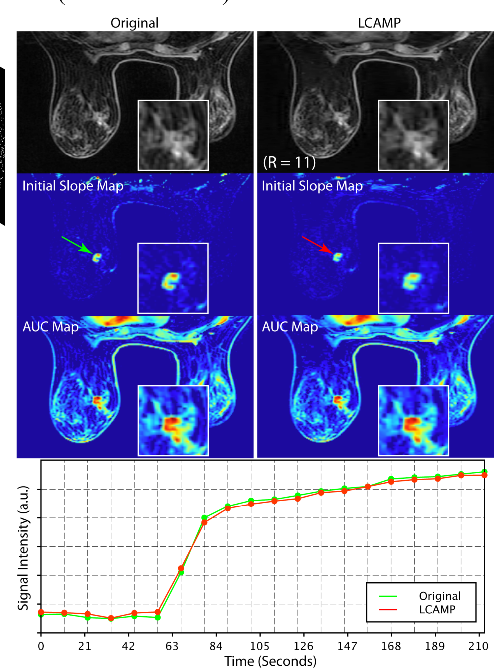

Fig 3: *Comparison of original DCE data and LCAMP.*

Conclusion: We have presented a novel CS reconstruction method called LCAMP (Location Constrained Approximate Message Passing). The reconstruction is extremely fast due to its low computational complexity, and robust due to its non-zero location assumption. LCAMP also can be more practical since there is no regularization parameter to adjust.

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Reference: [1] Donoho, IEEE Trans Inf Theory 1995;41(3):613, [2] Daubechies et al., Comm Pure Appl. Math 2004;57:1413, [3] Donoho, IEEE TIT, 2006;52(4):1289, [4] Donoho et al., PNAS, 2009;106:18914, [5] Chang et al., IEEE Trans Image Process. 2000;9(9):1532