High-Frequency Subband Compressed Sensing with ARC Parallel Imaging

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Introduction: Compressed sensing (CS) is an acquisition and reconstruction technique that can reduce the measurement size [1]. In this work, we present a novel way to efficiently combine CS and parallel imaging (PI) by separating the estimation methods in k-space. We apply CS to estimate high-frequency k-space data, and use ARC (Autocalibrated Reconstruction for Cartesian sampling) PI to estimate low-frequency k-space data. This exploits the wavelet characteristics where high-frequency wavelet subbands are typically the most sparse [2], and the ARC convolution kernel nature where regular undersampling provides more stable reconstruction. This method allows easy incorporation of the two (ARC followed by CS) and further removes possible CS failure in low-frequency region. This work has been demonstrated for high-resolution 3D breast imaging.

Theory: Wavelet transforms capture both k-space and image domain information by applying an iterated filter bank (high- and low-pass filters and downsampling). Fig 1a shows the relationship between high-frequency subbands ($w_{LH/HL/HH}$) and corresponding k-space data ($y_{LH/HL/HH}$). The Fourier transform (FT) of the wavelet (spectral weighting) determines how each filter bank localizes k-space, and the corresponding k-space data ($y_{LH/HL/HH}$) are approximately a multiplication of the spectral weighting and the FT of the upsampled wavelet subband [2]. When the spectral weighting is removed, the processed data bands ($u_{LH/HL/HH}$) are just an FT of each upsampled wavelet subband ($w_{LH2/HL2/HH2}$). Fig 1b shows the relationship between w_{LH} and y_{LH}.

The wavelet coefficients can be organized into a tree structure by the nature of the concatenated filter bank. The wavelet tree (Fig. 1a) shows that a wavelet coefficient is non-zero only if the wavelet coefficient in the upper scale (lower frequency) is non-zero (with very few exceptions) [3]. This wavelet-sparse relationship, therefore, provides two interesting observations: high-frequency subbands are the most sparse, and possible non-zero locations of high-frequency subbands are limited based on knowledge of the upper scale wavelet subbands.

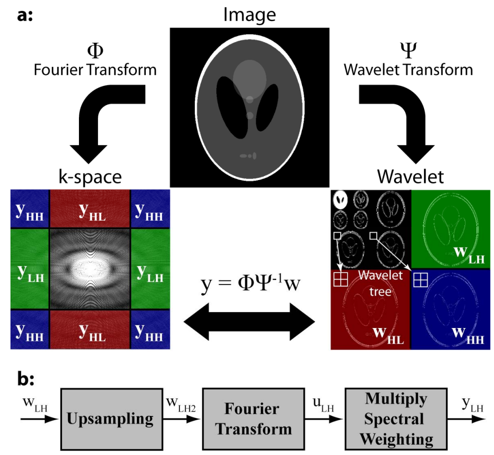

Fig 1: The relationship between Fourier (Φ) and wavelet (Ψ) transforms: (a) high-frequency subbands ($w_{LH/HL/HH}$) and k-space data ($y_{LH/HL/HH}$), and (b) the relationship between w_{LH} and y_{LH}.

Methods: Fig. 2 shows the k-space sampling (k_y-k_z) and the serial reconstruction of ARC and CS. The sampling pattern consists of three parts: a fully sampled region for coil calibration, a regularly undersampled region for ARC, and a randomly undersampled region for CS. Note that low-frequency contents are well recovered after ARC but high-frequency contents (e.g. fine structure of comb) are not recovered.

After ARC reconstruction, we divide each section of k-space ($y_{LH/HL/HH}$ in Fig 1a) by the spectral weighting. Each processed data section then becomes an FT of the wavelet subband and this creates three independent smaller-sized L_1 minimization problems:

$$\textbf{Minimize } |w_i|_1, \textbf{ s.t. } u_i = \Phi_m w_i \qquad i = \text{LH or HL or HH}$$

where w_i is wavelet coefficients in the high-frequency wavelet subband, u_i is k-space data after removing the spectral weighting, and Φ_m is a upsampling version of randomly undersampled FT. We named this newly derived minimization as **HiSub CS** (High-frequency Subband Compressed Sensing). An approximate message passing (AMP) algorithm [4] is used to solve the above equation. The AMP is a variation of the iterative thresholding method by adding a message passing term at iteration and is known to be faster and more adequate for large-sized problems [4]. We have added a non-zero location constraint estimated from the upper scale wavelet coefficients. The non-zero location constraint replaces a role of hard-thresholding and therefore there is no free parameter to adjust. We used the Cohen-Daubechies-Feauveau (CDF) 9/7 wavelet transform where the transform supports non-power of two matrix sizes.

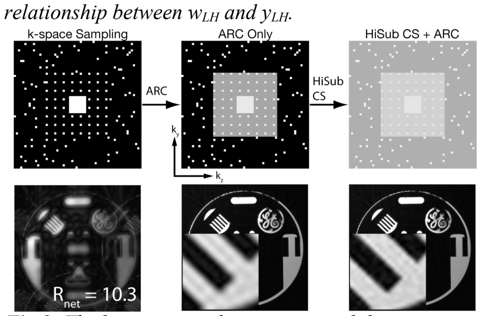

Fig 2: The k-space sampling pattern and the combination of ARC and HiSub CS.

Results and Discussion: Fig. 3 shows a representative reconstruction result for 3D breast MRI. Imaging experiments were performed on a 3.0T GE MR750 scanner. A 3D spoiled gradient echo sequence was used with a custom-fitted 16-channel breast array coil [5], enabling PI in two dimensions. The acquisition matrix size was 360×360×240. The acceleration factor for ARC PI (R_{PI}) is 3×2 (k_y×k_z), and the acceleration factor for CS (R_{CS}) is 16 in outer k-space. The net acceleration factor (R_{net}) is 10.8. The fully sampled and ARC (regularly undersampled at full resolution with R_{PI} = 3×2) data are shown as comparison. HiSub CS maintains fine structures (see arrows and zoomed images in Fig 3) and is close to fully sampled and ARC. Skipping k-space corners can further increase R_{net} to 12.8 without loss of overall image quality. CS only and ARC only with a similar level of acceleration have also been evaluated but they have suffered from severe residual artifacts (images not shown).

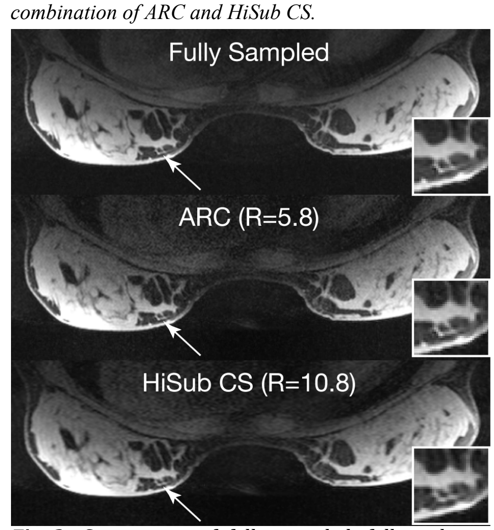

Fig 3: Comparison of fully sampled, full-resolution ARC (R = 5.8), and HiSub CS (R = 10.8) reconstruction.

Conclusion: The proposed method applies separate k-space sampling and reconstruction for high- and low-frequency k-space data by considering the link between k-space and wavelet domain. This eliminates possible CS reconstruction errors in the low-frequency k-space region. This reconstruction example showed HiSub CS successfully recovered low-frequency content (using PI ×6) and fine structures (using CS ×16) with a net acceleration of 10.8.

Reference: [1] Donoho, IEEE TIT, 2006;52(4):1289, [2] Candes et al., Inverse Problems, 2007;23:969 [3] Baraniuk, et al., ACM, 2002;16:357 [4] Donoho et al., PNAS, 2009;106:18914, [5] Nnewihe et al., 2010 ISMRM p644.