ESPIRiT (Efficient Eigenvector-Based L1SPIRiT) for Compressed Sensing Parallel Imaging - Theoretical Interpretation and Improved Robustness for Overlapped FOV Prescription

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Introduction: To achieve high acceleration needed for 3D volumetric MRI, recent efforts have integrated compressed sensing (CS) constraints in parallel imaging (PI) reconstruction. A representative approach among such efforts is L_1SPIRiT [1] that provides a flexible framework combining PI and CS synergistically and has demonstrated effectiveness in clinical evaluations [2]. A computationally efficient derivative of L_1SPIRiT (ESPIRiT) [3] lowered the computation complexity of its PI and CS operators based on eigenvector calculations. The purpose of this work was to analyze the similarities of the above two approaches from a theoretical viewpoint and thereby identify a method for improving the robustness of ESPIRiT, specifically its compatibility with overlapped FOV acquisition (or 'phase-wrap') commonly encountered in clinical imaging.

Theory: L_1SPIRiT utilizes coil sensitivity by iteratively enforcing self-consistency in k-space, namely, resynthesizing k-space by convolving the current k-space solution with a uniform SPIRiT convolution kernel (G_k) [1,4]. This operation, when equivalently applied in image domain, amounts to generating a new image (X_{n+1}) by multiplying the current solution (X_n) with a matrix G_I at each pixel: $X_{n+1}=G_IX_n$ (1), where $G_I=F^{-1}(G_k)$. This operation can be interpreted from the following two separate perspectives:

Perspective 1: mathematically, when iteratively applied as in L_1SPIRiT, the operation defined by Eq.1 resembles the power iteration algorithm for calculating the dominant eigenvector. When $n\text{->}\infty$, X_n linearly approaches the dominant eigenvector (v) of G_I, corresponding to the largest eigenvalue (λ) of G_I.

Perspective 2: in the context of MRI, the reconstruction should ideally converge to the truth image, X. In another word, G_I is an identity-transformation for X: $X=G_IX$ (2), where X equals the magnetization image (M) modulated by coil sensitivity (C). Considering the possibility of signal overlap caused by selection of an insufficiently large FOV either accidentally or intentionally for reducing scan time, X can be expressed in a general form as $X=\sum(M_mC_m)$ (3), where M_m represents magnetization aliased into the same pixel originating from different physical locations, C_m represents coil sensitivity at the corresponding locations. Combining (2) and (3) leads to $\sum(M_mC_m)=G_I\sum(M_mC_m)$ (4). Eq.4 indicates that X can be represented by linear combination of bases - the eigenvectors of G_I with $\lambda=1$: 1) at an air pixel without MR signals, the PI operator is ill-conditioned and λ could be much different from 1; 2) at a nonoverlapping pixel, there is a unique v equal to C [3]; 3) at a pixel with signal overlapping, algebraic multiplicity of $\lambda=1$ occurs, resulting in possibly multiple distinct v_m's with C_m as one possible set of such eigenvectors.

Combining perspective 1&2, we understand that, when $n\text{->}\infty$, X_n is aligned with the dominant eigenvectors (v_m's) of G_I and the largest eigenvalue is 1. Therefore, rather than performing the expensive matrix operation in Eq.1, we can alternatively enforce the PI constraint by directly projecting X_n to v_m's: $X_{n+1,m}=\sum((X_nv_m^H)v_m)$, if v_m's are orthogonal, where H is conjugate transpose. L_1SPIRiT with this operation should converge to the same solution as original L_1SPIRiT in pixels with MR signals [3]. Note that due to numerical precision limits, the computed v_m's may have slightly different λ's (~1.0). An example is shown in Fig.1. At frontal lobe where signal aliasing occurs, the second largest λ is also ~1.0, while in other pixels without aliasing, the value is much lower (max<0.6).

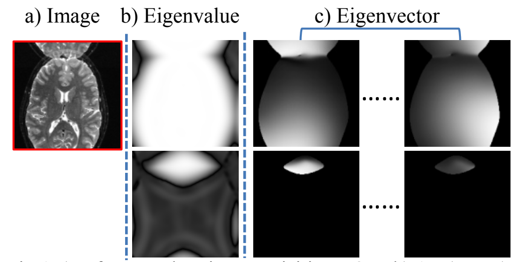

Fig.1 a) reference showing acquisition FOV, b) 1st (upper) & 2nd largest (lower) eigenvalue maps and c) their corresponding eigenvector maps for different coil channels

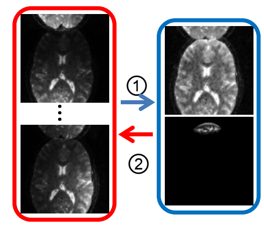

Fig.2 left: X; right: M

Methods & Materials: Based on the above analysis, original ESPIRiT taking into account a single v of the λ closest to 1 is sensitive to FOV overlap and therefore is modified to incorporate possible algebraic multiplicity of $\lambda=1$. The modifications include: 1. in calibration, we now calculate all v_m's of G_I with $\lambda\approx1$ (Fig.1.c) and then orthorgonalize v_m's in each pixel; 2. in iterations ,as shown in Fig.2, we calculate multiple magnetization images (M_m) from current coil images (X_n), $M_{n,m}=X_nv_m^H$ ((1) in Fig.2), and perform CS on them separately pursuing joint sparsity and next overlap them to reproduce a new solution (X_{n+1}) with signal aliasing, $X_{n+1}=\sum(M_{n,m}v_m)$ ((2) in Fig.2), to enforce consistency with acquired data.

To evaluate the modified ESPIRiT algorithm, full k-space was collected from 3 healthy volunteers with slight FOV overlap and undersampled offline using incoherent Poisson disk sampling to simulate accelerated acquisition (2×2 in [ky,kz]). The 3 datasets were acquired with standard protocols for 1. T2-weighted brain MRI, 2. whole-heart coronary MRA and 3. noncontrast-enhanced renal MRA on GE 1.5T MR scanners using 8-channel coils. Images were reconstructed using L_1SPIRiT and both modified and original ESPIRiT with 30 iterations.

Results: On all datasets, visible artifacts were observed on images reconstructed using original ESPIRiT, while modified ESPIRiT corrected such artifacts and provided image quality similar to L_1SPIRiT. The reconstruction time was about 8-15 mins for ESPIRiT, depending on imaging resolution, using Matlab running on an 8-processor platform. The mean reconstruction errors were 15.74%, 14.48% and 14.94% for original and modified ESPIRiT and L_1SPIRiT, respectively. Fig.3 shows a representative example from cardiac MRI, where overlap on chest wall is commonly used. We can notice that $\lambda\approx1$ (white) occurs on Fig.3.a in pixels with MR signals and on Fig.3.b in pixels with signal aliasing, e.g. chest wall with aliasing along y due to small FOV selection (solid arrow) and peripheral slices presumably due to signal aliasing from out-of-slab spins excited by side-lobes of RF excitation (dotted arrows). Original ESPIRiT generates remarkable artifacts propagating across the entire FOV (arrows in Fig.3.c). In comparison, the modified ESPIRiT image is free of such artifacts (Fig.3.e), similar to L_1SPIRiT (Fig.3.g). On the error images, we can more clearly see the improved accuracy on both the chest wall and the heart for the modified ESPIRiT reconstruction.

Conclusion: This work analyzed the convergence of L_1SPIRiT and ESPIRiT and demonstrated that in theory they should converge to the same solution. Also, we demonstrated the existence of multiple dominant eigenvectors for data acquisition with FOV overlap and showed that the original ESPIRiT approach including only one dominant eigenvector generates considerable artifacts, similar to mSENSE [5]. Such artifacts can be corrected by the proposed modified ESPIRiT. Our results based on invivo datasets confirmed these theoretical predictions. In conclusion, ESPIRiT can greatly reduce L_1SPIRiT computation without sacrificing image quality and is a promising solution to robust and autocalibrated compressed sensing parallel imaging reconstruction for highly accelerated MRI.

References: [1] Lustig, ISMRM, 2009:334; [2] Vasanawala, Radiology, 2010, 256:607; [3] Lai, ISMRM, 2010:345; [4] Lustig, MRM, 2010, 64:457; [5] Griswold, MRM 2004, 52:1118

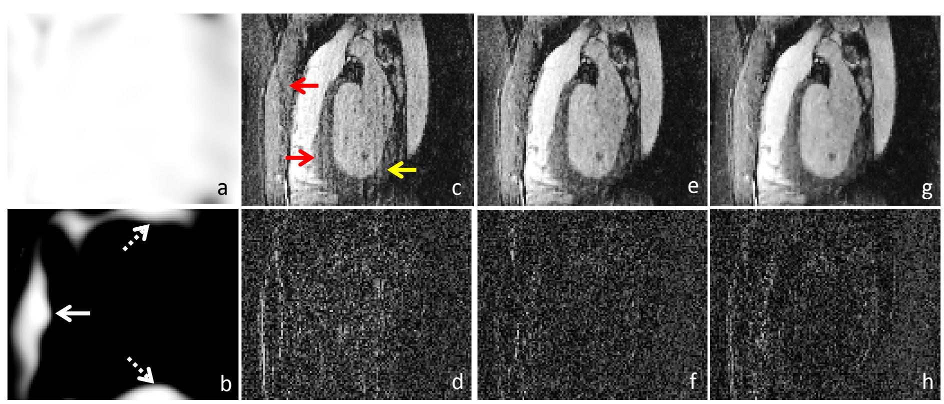

Fig.3 (horizontal:y, vertical:z) a) 1st & b) 2nd largest eigenvalue maps; images reconstructed using c) original & e) modified ESPIRiT and g) L1SPIRiT; d, f, h are error images of c, e, g. respectively.