Derivative Encoding for Parallel Imaging

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Introduction

It is well-known that coil sensitivity profiles can be characterized using spatial polynomial functions. For example, in SENSE (1), polynomial functions are used to fit coil sensitivity profiles for extrapolation and noise removal. According to the Fourier derivative theorem, the product between the weighted spin density and a spatial polynomial term is the Fourier transform of a k space partial derivative. In this work, we show that the partial derivatives with respect to the undersampled k space direction can be expressed as a linear combination of partial derivatives with respect to the perpendicular, fully sampled k space direction(s). This linear relationship between partial derivatives is shift-invariant in the Cartesian k space coordinate system. We demonstrate that, using autocalibrating signals (2), images can be unwrapped using partial derivatives. We also show that partial derivatives can reduce noise amplification in SENSE reconstruction.

Theory and Methods

For simplicity, we consider a two-dimensional slice. Its k space signal is sampled in the Cartesian coordinate with undersampling in the k_y direction only. By expressing coil sensitivity profile $c_{j'}(x,y)$ as a two-dimensional polynomial series it can be shown that

$$\frac{\partial s_{j'}^{n}(k_x,k_y)}{\partial k_y^{n}} = \sum_{j:\text{all coils}} \sum_{m=0}^{M-1} \varepsilon_{j'jmn} \frac{\partial s_{j}^{m}(k_x,k_y)}{\partial k_x^{m}} \quad [1]$$

That is, the partial derivatives of k space signal of coil j' with respect to k_y is a linear combination of 0-(M-1)th partial derivatives with respect to k_x summed over all coils. $\varepsilon_{j'jmn}$'s are shift-invariant coefficients. Using Fourier transform, Eq.[1] is equivalent to $\partial h_{j'}^{n}(x,k_y)/\partial k_y^{n} = \text{sum}_{j:\text{all coils}}\{\kappa_{jn}(x)h_j(x,k_y)\}$ [2]. According to the Fourier derivative theorem, $\mathcal{F}\{\partial h_{j'}^{n}(x,k_y)/\partial k_y^{n}\} = (-i2\pi y)^n c_{j'}(x,y)\rho(x,y)$. The Fourier partial derivative theorem indicates that aliased pixels generated from Fourier transform of partial derivatives contain information on their spatial origins in the aliased y direction, therefore, providing additional constraints for image unwrapping. These new constraints $y_1^n c_{j'}(x,y_1)\rho(x,y_1) + y_2^n c_{j'}(x,y_2)\rho(x,y_2) + \ldots (n\neq 0)$ can be used in conjunction with SENSE (n=0) for parallel imaging reconstruction in the image domain. Expanding $s_{j'}(k_x,k_y+n\Delta k_y)$ as a Taylor series around k_y, Eq.[1] becomes $s_{j'}(k_x,k_y+n\Delta k_y) = \text{sum}_{j:\text{all coils}}\{\text{sum}_{m:0\sim M-1}\{\lambda_{jj'mn}\partial s_j^{m}(k_x,k_y)/\partial k_x^{m}\}\}$ [3], which can synthesize skipped k space point $s_{j'}(k_x,k_y+n\Delta k_y)$ using $\partial s_j^{m}(k_x,k_y)/\partial k_x^{m}$.

Fully sampled MPRAGE brain images (256x256, slice thickness=1.3 mm), gradient and spin echo images (256x256, slice thickness=2-5mm) acquired at 3T with an 8-ch coil were used to test the partial derivative methods. Partial derivatives with respect to k_y were either directly measured using tightly spaced k_y lines and k space polynomial fitting or calculated using a fully sampled k space core. When a fully sampled core is used, we multiply $h_{j'}(k_x,y)$ with y^n and then Fourier transform the hybrid signal into $\partial s_{j'}^{n}(k_x,k_y)/\partial k_y^{n}$. Partial derivatives with respect to k_x were also calculated using Fourier transform except that the full k_x length was used. The shift invariant coefficients $\varepsilon_{j'jmn}$ (or $\kappa_{jn}(x)$, or $\lambda_{jj'mn}$) were calibrated using the fully sampled k space core in a fashion similar to GRAPPA (2) autocalibration. Then, these coefficients will be used to calculate $\partial s_{j'}^{n}(k_x,k_y)/\partial k_y^{n}$ (or $\partial h_{j'}^{n}(x,k_y)/\partial k_y^{n}$, or $s_{j'}(k_x,k_y+n\Delta k_y)$) for the undersampled k_y lines because, according to Eq. [1], partial derivatives with respect to k_y are fully determined by information obtained from the same k_y lines acquired by all the coils.

Results and Discussion

As a proof of concept, a gradient echo phantom image was reconstructed in k space using half of the k_y lines + 20 ACS lines for R=2. Partial derivatives $\partial h_{j'}^{n}(x,k_y)/\partial k_y^{n}$ for No. 63, 64, 65, 66, and 67 k_y lines were measured by fitting the five tightly spaced k_y points at each $h_{j'}(x,k_y)$ coordinate to a third order polynomial. Two sets of aliased images were obtained: a conventional one described by $[c_{j'}(x,y_1)\rho(x,y_1) + c_{j'}(x,y_2)\rho(x,y_2)]$ and a new one described by $[y_1c_{j'}(x,y_1)\rho(x,y_1) + y_2c_{j'}(x,y_2)\rho(x,y_2)]$. The unaliased individual coil images were thus reconstructed. There are no visible reconstruction artifacts in the unwrapped individual coil images (data not shown), demonstrating that y-ordinates (y^n) can be used as independent constraints for imaging unwrapping.

Fig. 1. By Eq.[3]

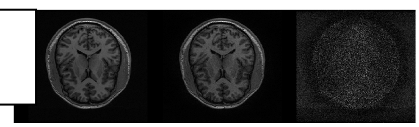


Table 1. Relative m.s.e. (R=4)

Images	GRAPPA	Eq.[3]
MPRAGE	2.3×10^{-2}	1.8×10^{-2}
grad. echo	2.8×10^{-3}	2.2×10^{-3}
spin echo	1.8×10^{-2}	1.2×10^{-2}
phantom	2.2×10^{-4}	1.5×10^{-4}

We compared k space image reconstruction using Eq. [3] (R = 4, 24 ACS lines, M=12) with a standard GRAPPA routine (www.nmr.mgh.harvard.edu /~fhlin developed by Dr F.H. Lin at MGH, R = 4, 25 ACS lines). For each skipped k space point, the partial derivatives with respect to k_x of its two immediate undersampled neighbors are linearly recombined to synthesize it. Typical relative mean square errors (m.s.e.) between fully sampled sum-of-square images and the reconstructed images were listed in Table 1. The fully sampled MPRAGE image (left), accelerated image reconstructed by Eq.[3] (middle) and the difference image (x8, right) were shown in Fig. 1. Since calculation of derivatives using FFT only adds a minimal computational load the speed of image reconstruction by Eq.[3] is comparable to that of GRAPPA. The results in Table 1 also show that k space image reconstruction using partial derivatives may lead to reduced reconstruction error.

Fig. 2. By SENSE (+derivatives)

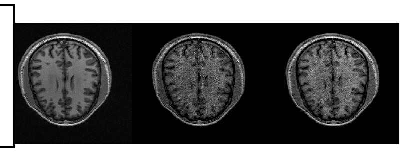
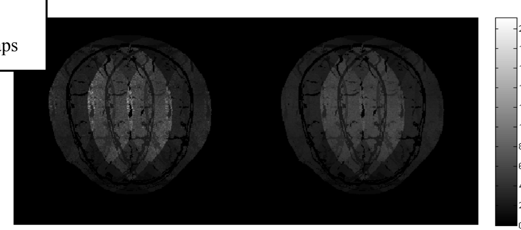


Fig. 3. g-fac maps



When accurate sensitivity maps are obtainable and only 1/R evenly spaced k_y lines are acquired, SENSE reconstruction gives optimal results. For high R/No. of coils ratios, however, SENSE leads to very large g factors due to ill-conditioning of its encoding matrices. Here we show that it is possible to improve the conditioning of SENSE encoding matrices by adding additional constraints derived from partial derivatives encoding (i.e., $y_1^n c_{j'}(x,y_1)\rho(x,y_1) + y_2^n c_{j'}(x,y_2)\rho(x,y_2) + \ldots (n\neq 0)$). The derivative constraints are different from regularization, the latter leads to loss in spatial resolution. Fig. 2(left) shows a fully sampled sum-of-square MPRAGE image. The fully sampled individual coil images were used to calculate sensitivity profiles. Accelerated image reconstructed using SENSE (R=6, middle) shows large noise amplification as expected. Its g factor map is given by Fig. 3(left). For comparison, accelerated image reconstructed using SENSE + derivative constraints $[y_1c_{j'}(x,y_1)\rho(x,y_1) + \ldots + y_6c_{j'}(x,y_6)\rho(x,y_6)]$ (R=6, right, 20 ACS lines) was shown in Fig. 2(right). Its g factor map was shown in Fig. 3(right). For the SENSE + constraints image (Fig. 2(right)), Eq.[2] (n=1) was used for autocalibration and calculation of partial derivatives for the undersampled k_y lines because Eq.[2] allows for easier tracking of noise propagation. Each constraint $[y_1c_{j'}(x,y_1)\rho(x,y_1) + \ldots + y_6c_{j'}(x,y_6)\rho(x,y_6)]$ was multiplied by a weighting factor (0.5) found empirically which gives approximately the maximum reduction of the largest SENSE g factor (from 21 (SENSE) to 8.4 (SENSE+derivative constraints). In both Fig 2(right) and Fig. 3(right), the additional constraints derived from derivative encoding significantly improved the quality of the accelerated image without any loss in spatial resolution (the cost of acquiring the 20 ACS lines was not considered). Future work will use constraints derived from higher order partial derivatives (n>1) and optimized, spatially dependent weighting factors.

References: 1. Pruessmann et al, Magn Reson Med **42**, 952-962 (1999). 2. Griswold et al, Magn Reson Med **47**, 1202-1210 (2002).