# Post-Cartesian Calibrationless Parallel Imaging Reconstruction by Structured Low-Rank Matrix Completion

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**Introduction:** Previously, we presented a new autocalibrating parallel imaging (acPI) method that does not explicitly require autocalibration lines and is based on low-rank matrix completion [7]. This time, we extend this technique to non-Cartesian trajectories. Our method jointly autocalibrates and reconstructs the images, similarly to other joint estimation (coil & data) techniques [2-5]. However, the latter still require some calibration or coil estimates.

**Theory:** ***Low-rank matrix completion*** is a hot research topic and is an extension of compressed sensing to matrices [5]. In general, missing entries of a matrix can be completed if the original matrix has a low-rank and incoherence conditions (randomly undersampled entries) exist. Efficient algorithms for reconstruction are based on singular-value thresholding [5], which we use here.

***acPI as low-rank structured matrix completion:*** GRAPPPA and other acPI methods [1,4,6] exploit linear dependency in k-space. Overlapping blocks in k-space (across coils) are linearly dependent, which enables calibration of GRAPPA interpolation kernels. This means that a Calibration matrix, in which the rows are made of data from overlapping blocks in k-space has low-rank (see fig. 1). Therefore, an (incoherently) undersampled k-space acquisition can be recovered by completing missing entries, which give the lowest rank matrix. Here, because our original data is non-Cartesian and does not lie on a grid, we search for a low-rank matrix which when interpolated to non-Cartesian grid is close to the acquired data. The following is a very efficient algorithm based on singular-values thresholding [6] (Fig. 2): (1) Construct matrix $A$ from overlapping blocks (2) Compute $[U,\Sigma,V]$ = svd($A$); Threshold the singular values $\Sigma = S(\Sigma,\lambda)$; (3) Compute: $A = U\Sigma V'$ (4) Reconstruct k-space, from $A$. (5) Impose data consistency by computing the residual on a non-Cartesian grid, grid it and add to the current estimate. (6) Repeat 1-5 till convergence.

**Methods and Results:** We acquired data using a 60 interleave spiral acquisition with 30cm FOV and 0.75mm resolution. The readout length was 5ms. We reconstructed images from 20/60 interleaves (3-fold acceleration) using gridding, SPIRiT[6] and the proposed method. For SPIRiT we used the full data to perform calibration. For the proposed method, overlapping window size was 6x6x8, hard singular value threshold function was set to choosing the largest 45 (of 288) singular values. The number of iterations was N=30. The results are demonstrated in Fig. 3 showing a similar good reconstruction as SPIRiT, but with autocalibration from undersampled data only.

**Conclusions:** We demonstrated a truly non-Cartesian auto-calibrating PI method based on low-rank matrix completion, which is able to reconstruct images with no oversampled area in k-space. It can be used with arbitrary trajectories and produces images with quality similar to those obtained with calibration.

**References:** [1] Griswold et. al MRM 2002;47(6): 1202-10 [2] M. Uecker et. al MRM 2008:60:674-682 [3] Ying et. al MRM 2007:57:1196-1202 [4] Zhao et. al MRM 2008:59:903-7 [5] Cai, et.al "A singular value thresholding algorithm for matrix completion." 2008, on line manuscript. [6] Lustig et. al MRM 2010;64(2):457-7 [7] Lustig et. al, ISMRM'10 pp. 2870

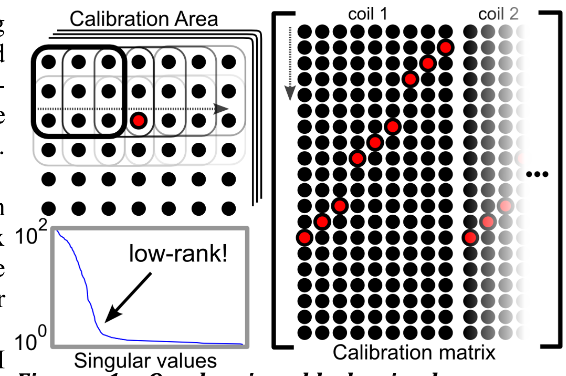


***Figure 1: Overlapping blocks in k-space are linearly dependent; therefore a matrix in which rows are made of overlapping blocks has low-rank. It also has a Hankel structure (illustrated by red circles). This further reduces the degrees of freedom in a completion problem.***

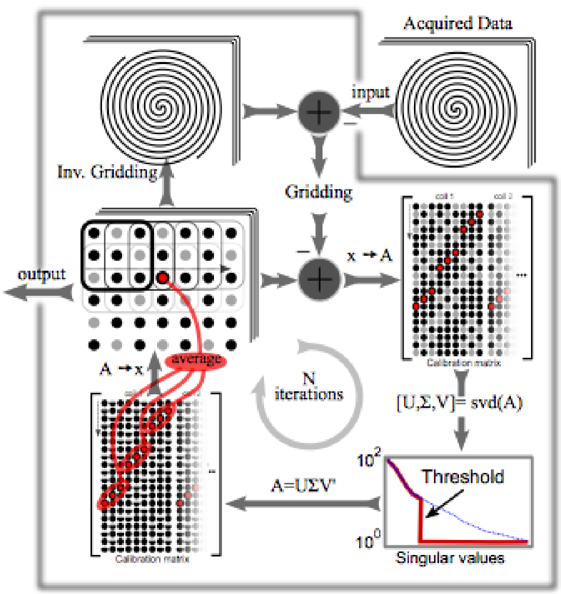


***Figure 2: Non-Cartesian autocalibration. The current estimate is reordered into a calibration matrix. Low-rank is imposed by singular value thresholding. Hankel structure is imposed averaging multiple entries. Data acquisition consistency is imposed by computing the residual on a non-Cartesian grid, gridding it and subtracting from the current estimate.***

***Figure 3: Reconstruction from 3-fold undersampled spiral trajectory. Left: gridding. Middle: SPIRiT with separately calibrated kernel. Right: The proposed reconstruction achives the same quality as SPIRiT, only without any extra calibration information.***