A performance measure for MRI with nonlinear encoding fields

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INTRODUCTION MR imaging is conventionally performed using gradient fields that vary linearly across the field-of-view. Interest in nonlinear magnetic encoding fields has recently been reinvigorated with the introduction of two parallel imaging techniques that utilise nonlinear gradients, PatLoc [1,2] and O-Space [3]. With the advent of these techniques, comes the need to quantify the performance of general encoding schemes efficiently and accurately.

PatLoc employs nonlinear non-bijective fields, and similar to conventional SENSE imaging [4], the additional encoding inherent in the receive coils is crucial to resolving the ambiguity associated with non-bijectivity. With a carefully chosen non-uniform pixel basis, PatLoc can be viewed as a generalisation of SENSE for nonlinear fields, thus inheriting the analytical results concerning SNR and g-factor, with a correction for the non-uniform pixel basis [2]. For other acquisition schemes that do not have this exploitable structure, a PatLoc-like performance analysis is not straight-forward. For example, O-Space imaging utilises a nonlinear encoding field that is spatially translated between echoes, and consequently the notion of k-space cannot be defined. Computing the variance of the reconstructed pixel by inverting the acquisition matrix is impractical for reasonably sized images. It is therefore difficult to quantify the performance of arbitrary encoding schemes.

The mathematical theory of frames has been applied to MRI fields of compressive sensing and wavelet encoding [5]. In this work, we apply frames in the context of parallel imaging using nonlinear encoding fields to develop a quantitative measure of performance that is both computationally efficient and universally applicable to any given nonlinear schemes.

THEORY The object encoding functions form the frame elements and consequently define the frame operator, Eq.(1). The frame operator captures all properties of the encoding scheme; the point-spread function, reconstruction noise, and truncation effects can all be derived from the frame operator. Since we ultimately aim to reconstruct a finite dimensional image from the measurements, space is discretised according to a pixel basis, $\chi_p(\mathbf{x})$, usually Dirac delta distributions. Under discretisation, the frame operator becomes a matrix with elements defined by Eq.(2).

$$(Sf)(\mathbf{x}) = \sum_{l,k,i} \int f(\mathbf{z}) c_l(\mathbf{x}) c_l(\mathbf{z}) e^{j(\phi_{k,i}(\mathbf{x}) - \phi_{k,i}(\mathbf{z}))} d\mathbf{z} \qquad (1)$$

$$S_{pq} = \sum_{l,k,i} \iint \chi_q(\mathbf{x}) \chi_p(\mathbf{z}) c_l(\mathbf{x}) c_l(\mathbf{z}) e^{j(\phi_{k,i}(\mathbf{x}) - \phi_{k,i}(\mathbf{z}))} d\mathbf{z} d\mathbf{x} \qquad (2)$$

$$m_p = \frac{\sum_q |S_{pq}|}{S_{pp}} \qquad (3)$$

In the above expressions, $c_l(\mathbf{x})$ is the coil sensitivity for the l^{th} coil and $\phi_{k,i}(\mathbf{x})$ is the accumulated phase for the k^{th} echo and i^{th} time sample. For simplicity, susceptibility and relaxation effects are ignored. In the case of conventional Fourier imaging the frame matrix is diagonal, whereas for both SENSE and PatLoc, a pixel basis can be chosen such that the matrix is block diagonal and the invertibility analysis is simplified to the analysis of individual blocks. No such basis can be chosen for other techniques such as O-space imaging. Instead, we define the simple yet powerful metric in Eq.(3), comparable to the width of the point-spread-function.

Applied to any encoding scheme, the proposed metric in Eq.(3) highlights regions that exhibit poor resolvability. This is analogous to the covariance matrix in SENSE encoding which identifies areas in the image with large reconstruction variance. The metric is also related to the Gerschgorin circle theorem concerning the eigenvalues of the frame matrix, from which the matrix condition number can be defined.

METHODS The universal applicability of the proposed performance metric to PatLoc, O-Space and SENSE techniques is demonstrated through simulated data, as follows. Coil sensitivity profiles were generated according to the Biot-Savart equation for eight receive coils circumferentially distributed around the field-of-view. To validate the qualitative behaviour of the performance map, we compared our map to a 64×64 reconstruction variance map computed by full inversion of the frame matrix, with elements as per Eq.(2). PatLoc, SENSE and O-space define their own $\phi_{k,i}(\mathbf{x})$ as follows. **PatLoc:** A radially quadratic field was used for the readout field and a quadrapolar field, with sinusoidal variation in azimuth was used for phase encoding, as in [1]. **O-Space:** A quadratic field with 64 different centre points arranged in concentric rings was simulated, with 256 time samples, as in [3]. **SENSE:** The frame operator for SENSE was constructed using 4-fold acceleration in the phase direction. In all cases, FOV=10cm × 10cm, and a total of 64 echoes were simulated. The proposed performance metric, Eq.(3), was applied with the same imaging parameters to generate maps of each imaging scheme's performance. In order to demonstrate the efficacy of the proposed performance measure, an O-Space construction, similar to [6], was formulated in which ten different centre placement schemes were evaluated, using the metric $C = \sum_\nu m_\nu$. This performance analysis is computationally tractable, avoiding the need to compute the reconstruction.

RESULTS The proposed performance metric, Eq.(3), captures the qualitative behaviour of reconstruction performance for each of the PatLoc, SENSE and O-Space imaging techniques (Fig.1). It is evident that the predicted reconstruction error for O-Space is more uniform than PatLoc, which exhibits the characteristic "hole" in the centre. Fig. 2 displays the performance of the ten centre placement schemes, and the performance map for each of the best and worst schemes, confirming the results of [6] that concentric rings is an optimised coil arrangement. Importantly, this result was achieved without performing the cumbersome inversion associated with reconstruction or variance analysis.

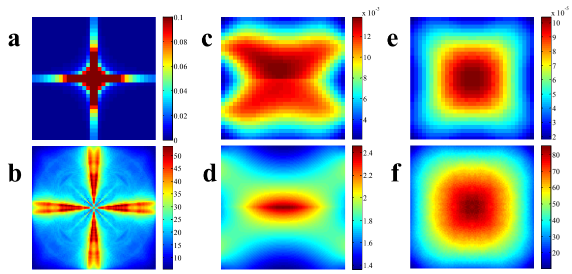

Figure 1: Top row: Reconstruction performance via matrix inversion. **Bottom row:** Reconstruction performance via proposed metric, Eq.(3). (**a**,**b**) PatLoc (**c**,**d**) SENSE (**e**,**f**) O-Space.

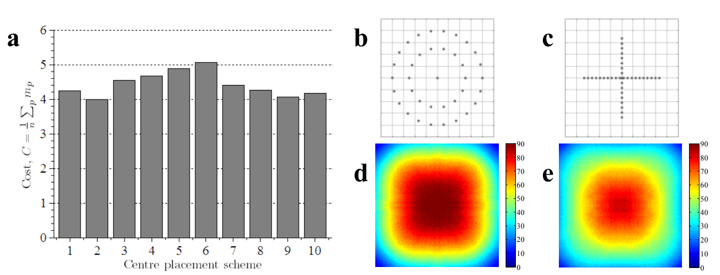

Figure 2: (**a**) The performance of ten O-Space centre placement schemes using $C = \sum_\nu m_\nu$. Corresponding centre placements for (**b**) best and (**c**) worst schemes. (**d**,**e**) Pixel-wise performance maps (Eq.(3)) of (**b**,**c**), respectively.

CONCLUSION We have formulated the problem of image reconstruction for any arbitrary encoding field by defining a frame and associated frame operator. The matrix form of the frame operator leads to the proposal of a simple but powerful metric to quantify the reconstruction performance. This metric is of particular benefit for techniques such as O-Space in which the frame operator does not exhibit an exploitable structure, and therefore no simple g-factor or modification thereof exists. Intuitively, our proposed performance metric provides a measure of how close the frame matrix is to diagonal, where an entirely diagonal matrix represents an optimal imaging scheme. Future work, will involve the use of this metric to optimise other imaging parameters such as coil positioning. The ability to quantify the performance of novel imaging techniques utilising nonlinear encoding fields is crucial for advancing research in this area.

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