K-Space Based Image Reconstruction of MRI Data Encoded with Ambiguous Gradient Fields

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Introduction: Recently, it has been shown that the application of non-bijective encoding fields (PatLoc [1], Fig. 1) results in aliasing artifacts similar to the fold-over artifact of standard accelerated parallel imaging (sPI). (Fig. 2c and 2h). From a mathematical point of view the artifacts are equivalent and therefore SENSE [2] can be applied for image unfolding [3] (Fig. 2). This is also possible when the PatLoc data are accelerated themselves (Fig. 2i,j). Aliasing resulting from the ambiguities of the encoding fields and from undersampling *can* be treated simultaneously with SENSE. However, they also *must* be treated simultaneously involving the inversion of a single combined sensitivity matrix.

In this abstract, we demonstrate that this behavior is fundamentally different with k-space based methods like GRAPPA [4]. The aliasing artifacts from field ambiguities and from undersampling can be separated from each other. It is shown that this property is essential for k-space based PatLoc image reconstruction.

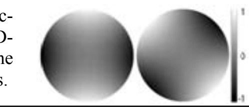
Theory: The key point of this abstract can be understood by considering the analogy of twofold accelerated PatLoc imaging with quadrupolar field encoding (Fig. 1) and sPI with acceleration factor 4 (Fig. 2d,i). In GRAPPA, calibration lines are acquired from which weights are determined. Consider now a sPI situation, where only twofold undersampled calibration lines have been acquired. It is then not possible to fully reconstruct the image, but at least they can be used to reconstruct intermediate coil images with only twofold aliasing. This is possible because the determination of the GRAPPA weights n is equivalent to minimizing the l_2-norm of the following expression:

$$\left\| \hat{c}_\alpha(\vec{k}) - \sum_{\{l,b_x,b_y\}=1}^{\{N_c,A_x,A_y\}} n(\alpha,l,b_x,b_y)\,\hat{c}_l\left(\vec{k}-(b_x\vec{e}_x - b_y\vec{e}_y)\Delta k\right)\right\|$$

This equation implies that the weights are found by approximating the k-space versions $\hat{c}_\alpha(\vec{k})$ of the different rf-coil sensitivities by a weighted sum of all coil sensitivities evaluated at the different sites of the GRAPPA kernel of size $A_x \times A_y$. As long as the k-space foot-print of the coil sensitivities significally exceeds Δk, this approximation also gives valid results for other GRAPPA operators or even sub-GRAPPA operators $B_x \times B_y \subset A_x \times A_y$. There is no reason why this approximation should fail when only undersampled calibration lines are available (simply corresponding to an increased Δk in the above equation). It is clear however, that the usage of undersampled calibration lines can only result in a partially unfolded image (Fig. 3, right). The crucial point about PatLoc is (within the analogy to sPI) that only "undersampled" calibration lines are available (Fig. 2h). These lines are sufficient to fill *PatLoc k-space*. Standard PatLoc reconstruction methods can then be used to finally reconstruct the image as full sampling still leads to aliasing due to the ambiguous encoding.

Methods: Reconstruction and simulations were programmed using MatLab (The Mathworks, Natick, USA). For the simulations, fourfold undersampled sPI was assumed with rf-coil sensitivities mimicking an eight-channel real world coil array. White noise was added to the coil data. Reconstruction was performed in two ways as shown in Fig. 3 (detailed description in the figure caption). In vivo PatLoc imaging data were acquired on a 3T Tim Trio (Siemens, Germany) with a PatLoc-coil insert [5] for fields as shown in Fig. 1. A twofold undersampled Cartesian PatLoc-128x256 dataset was acquired with 12 additional calibration lines. Similar to the simulations, the images were reconstructed in two steps: First, GRAPPA was applied to the undersampled dataset. Then, Cartesian PatLoc reconstruction [3] was applied (Fig. 4).

Figure1: Orthogonal multi-polar PatLoc-encoding fields of second order for 2D-imaging. These fields replace the standard x-gradient and y-gradient fields.



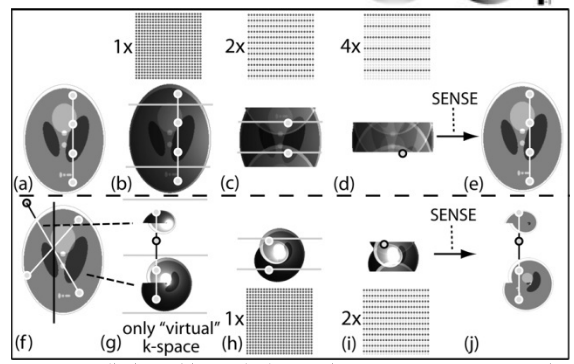


Figure 2: Aliasing with standard PI and PatLoc. Top row: Increasing acceleration leads to increased aliasing. The image can be unfolded using SENSE. Bottom row: The situation is similar in PatLoc. The distributed aliased image locations visible in (f) are equidistantly distributed in PatLoc encoding space (g) corresponding to no physical k-space. The ambiguities of the encoding field correspond to twofold undersampling (h) although in reality *PatLoc k-space* is fully sampled. Undersampling the PatLoc data leads to increased aliasing (i) , which can be undone using SENSE (j).

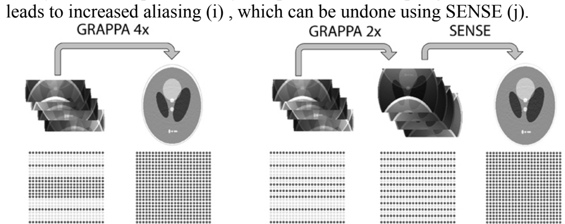


Figure 3: Reconstruction methods for undersampled sPI data. Left: All calibration lines are used to calculate the weighting factors for the individual missing lines. Right: Only each second calibration line is used to calculate half of the missing lines. The remaining aliasing is unfolded using SENSE.

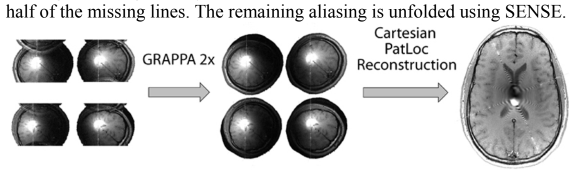


Figure 4: Proposed k-space based reconstruction for PatLoc. The undersampled k-space with acquired calibration lines is completed using standard GRAPPA. The resulting image is free of the undersampling artifact. Residual aliasing is due to the non-bijectiveness of the encoding fields. It can be unfolded using Cartesian PatLoc reconstruction.

Results: Fig. 3 indicates that GRAPPA with subsampled calibration lines and subsequent SENSE reconstruction gives almost the same results as direct GRAPPA reconstruction with fully sampled calibration lines. Fig. 4 demonstrates that a similar reconstruction - GRAPPA and subsequent Cartesian PatLoc reconstruction - also produces high-quality PatLoc images.

Discussion: The simulations confirm the theoretical considerations: Whereas in SENSE the reconstruction must be performed in one step, with GRAPPA, it is possible to reconstruct in subsequent steps with partial intermediate image unfolding. As a result of the analogy of accelerated PatLoc with highly accelerated sPI, this principle can be adopted to PatLoc imaging, where calibration lines are only available to partially unfold the image. In a second step, Cartesian PatLoc reconstruction can be applied to finally reconstruct the desired image. Other methods might be considered to reconstruct undersampled PatLoc datasets. The missing calibration lines of the fully sampled "virtual" PatLoc k-space (Fig. 2g) could for example be estimated using the coil sensitivity information, which is available in PatLoc imaging in any case. It would, however,be advantageous to use the above equation or modifications thereof directly to compute the desired GRAPPA weights because in this case, no calibration lines would be required at all. The possibility to use GRAPPA for undersampled PatLoc imaging data is encouraging in itself; however, it also suggests that other accelerated k-space trajectories can be handled by combining established k-space based reconstruction methods (like e.g. GROG [6]) with methods used for fully sampled PatLoc imaging data.

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References: [1] Hennig et al., MAGMA 21(1-2):5–14, 2008; [2] Pruessmann et al., MRM 42:952 –62, 1999; [3] Schultz et al., MRM 64:1390 – 1403, 2010; [4] Griswold et al., MRM 47:1002–10, 2002; [5] Welz et al., #762, ISMRM 2009; [6] Seiberlich et al., MRM 58:1257-65, 2007.