### Multi-dimensional encoded (MDE) magnetic resonance imaging

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#### INTRODUCTION

Magnetic resonance imaging has commonly used 3 orthogonal linear gradients to spatially encode objects in three dimensions. Recently, it has been demonstrated that images can be encoded and reconstructed efficiently by non-uniform spatial encoding magnetic fields (SEMs) using parallel acquisition with the localized gradient (PatLoc) system [1,2] or the O-space approach [3]. The flexibility of multiple SEMs in both PatLoc and O-space imaging suggests that it is possible to spatially encode an n-dimensional object in a m-dimensional space with m > n. Compared to acquisitions using only linear gradients, we can thus have a higher degree of freedom in encoding an object using spatial bases generated from the combination of different SEMs and even different RF channels in a coil array. Here we propose the multi-dimensional encoded (MDE) MRI using over-complete spatial bases with examples combining PatLoc and O-space imaging to achieve efficient encoding and image reconstructions with improved spatiotemporal resolution.

#### **METHOD**

The MDE MRI signal equation describes measuring an object with density  $\ \rho(\bar{r})$  using an RF coil with sensitivity  $\ c_r(\bar{r})$  in a system with multiple SEMs

$$G_{i}(\vec{r}), \ \textit{i=1...p}, \ \text{in a parametric data grid:} \\ s(r,q,j) = \int c_{r}(\vec{r}) \rho(\vec{r}) \exp \left(-2\pi y \left(k(q,j) \sum_{i=1}^{n_{G}} \alpha_{q,i}^{0} G_{i}(\vec{r})\right)\right) d\vec{r}, \quad r = 1 \cdots n_{C}, \ q = 1 \cdots p, \ j = 1 \cdots n_{E}(q); \ k(q,j) = k^{0}(q) \Delta t(q,j)$$

 $\Delta t(q,j)$  represents the lapse time at data grid j using the  $q^{th}$  SEM generated by driving each gradient element using a relative weight  $\alpha^0_{q,i}$  individually and a common scaling of  $k^0(q)$ .  $n_c$  and p indicates the number of RF channels and the number of SEMs.  $n_G$  is the nubmer of surface gradient element. Essentially, an object is now spatially encoded to a p-dimensional space.  $n_E(q)$  indicates the number of data collected using the  $q^{th}$  SEM. This equation can be transformed to a matrix representation to facilitate numerical implementation:  $\mathbf{s} = \mathbf{E} \, \mathbf{p}$ , where  $\mathbf{s}$  is the measurement vector and  $\mathbf{p}$  is the image to be reconstructed. Each row of the encoding matrix  $\mathbf{E}$  represents a spatial basis function generated from the combination of one RF coil sensitivity profile and one SEM. Note that conventional MRI has p=3 by three orthogonal linear gradients, while, for example, an 8-channel PatLoc system can have maximally p=3 to spatially encode a 3D object. The goal of MDE is to optimize  $\alpha^0_{q,i}$  and  $\Delta t(q,j)$  in conjunction with  $c_{p,i}(\vec{r})$  to efficiently encode p with a minimal measurements.

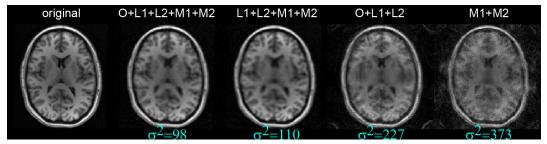
We used the greedy matching pursuit algorithm [4] to look for encoding coordinates for given p SEMs. Speficially, given a p-dimensional encoding coordinate, we examined encoding coordinates away from the current coordinate at 3 random positions in each encoding dimension following an exponential distribution ( $\lambda$ =10). The basis accounting for the most of the residual image variance among  $3^p$  choices was recruited in the dictionary. The image was replaced by the residual variance using all selected bases. Depending on the targeted acceleration rate, this procedure was repeated until the desired number of bases were selected. Given the optimal bases, we used iterative time-domain reconstruction to implement the conjugated gradient algorithm to complete image reconstruction [5].

We used Biot-Savart's law to simulate the  $B_1$  sensitivity maps and the spatial encoding magnetic fields (SEMs) of an **8-channel PatLoc** system with an **8-channel RF coil** array (**Figure at right**). Using singular value decomposition (SVD), the top five SEMs in this PatLoc system are one

right). Using singular value decomposition (SVD), the top five SEMs in this PatLoc system are one O-space SEM, two orthogonal linear SEMs, and two multipolar SEMs [5]. We specifically studied the **8-fold acceleration** MDE acquisitions and reconstructions using multipolar SEMs, multipolar and linear SEMs, O-space SEMs, and the combination of all five SEMs.

### **RESULTS**

Reconstructed images with 8-fold acceleration using two multipolar (M1, M2), two linear (L1, L2), and O-space imaging (O) SEMs were shown in the figure below. The residual variance of the reconstruction  $(\sigma^2)$  monotonically decreased as more SEMs became available.



# DISCUSSION

The MDE MRI presented here suggests that a more efficient spatial encoding can be achieved by a higher degree of freedom in basis functions realized by the combination of multiple RF sensitivity profiles and spatial encoding magnetic fields. Since the MDE image equation is still a linear equation, noise amplification can be calculated analytically. Our sampling scheme for the encoding coordinate is only one possibility among an intractable number of ways of selecting the spatial basis from an over-complete dictionary. A systematic procedure to optimize the encoding coordinates is desired to further improve the stability and the spatiotemporal acceleration of MDE MRI.

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