An Eigen-Vector Approach to AutoCalibrating Parallel MRI, Where SENSE Meets GRAPPA

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Introduction: Parallel imaging techniques can be categorized roughly into two families: explicit sensitivity based methods like SENSE[1] and autocalibrating methods (acPI) like GRAPPA[2]. In this work we finally bridge the gap between these approaches. We present a new way to compute the explicit sensitivity maps that are (implicitly) used by acPI methods. These are found by Eigen-vector analysis of the k-space filtering in acPI algorithms. Our Eigen approach performs like other acPI methods when the prescribed FOV is smaller than the object, i.e., is not susceptible as SENSE to FOV limitations [3]. At the same time, the reconstruction performs optimal calibration and optimal reconstruction, as SENSE. Our approach can be used to find the explicit sensitivity maps of any acPI method from its k-space kernels. For the sake of space we only describe our approach, which estimates them directly from the calibration matrix.

Theory: GRAPPPA and other acPI methods like SPIRiT [4] exploit linear dependency in k-space. Overlapping blocks in k-space (across coils) are linearly dependent, which enables the calibration of GRAPPA interpolation kernels. This is done by constructing a calibration matrix, *A,* and fitting the kernel, g, to the data, *y,* by solving a least-norm-least-squares problem Ag=y.
The linear dependence originates from the physics that there is a single source image imaged by different smooth coil sensitivities. Instead of performing calibration, by computing the SVD, $A=U\Sigma V^H$ we can find the support of the multi-coil data directly. Figure 1 shows that V can be clearly separated to $V_\parallel$ which spans the data, and $V_\perp$ which is orthogonal to it. The reconstruction from undersampled data is then finding the missing entries for which each block in k-space is supported by $V_\parallel$ and not by $V_\perp$. A similar idea was used in [5,6]. The reconstruction therefore must satisfy $x=V_\parallel V_\parallel^H x$, where $V_\parallel$ is the aggregated operation on all the blocks in k-space. However, since data originates from a single source multiplied by coil sensitivities functions, *S*, it should also satisfy $S=V_\parallel V_\parallel^H S$. The effective sensitivities are eigen-vectors of $V_\parallel V_\parallel^H$ with eigen-values '1'. These can be explicitly calculated directly by eigen-value decomposition in the image domain (similarly as in [7]). Figure 2 shows eigen-vectors in image domain that were calculated from 8-channel brain data. Indeed, the eigen-vectors with eigen-values `1` appear to be sensitivity maps! The eigen-vectors with eigen-value `1` can be used as sensitivities in any SENSE reconstruction.
A very interesting case is when the prescribed FOV is smaller than the object. Self calibrating SENSE methods, like mSENSE fail to reconstruct but GRAPPA works as expected. Figure 3 shows such a situation. In this case, there are more than one eigen-value '1' at each aliased pixel, with other set of sensitivities. acPI methods work in this case, since they explicitly use these extra sensitivities.

Methods and Results: 2-fold undersampled data sets were acquired using an 8-channel head coil, once with FOV smaller than the object in the phase encode. Eigen-value maps were calculated from 20x20 autocalibration lines by: 1) Constructing a calibration matrix, *A*, with [6x6] overlapping blocks 2) Computing $V_\parallel$ by choosing the largest 55/288 right singular vectors of *A* 3) Calculating the eigen-vectors for each pixel in image domain of $V_\parallel V_\parallel^H$ 4) The calculated maps, *S*, are eigen-vectors with eigen-value >0.98. Data was reconstructed with POCSENSE [8] using eigen-value maps, mSENSE[9] maps and with GRAPPA. The results in Fig. 4 show that our method has similar properties to GRAPPA, even though it was implemented with POCSENSE. Our reconstruction has lightly less noise in this case. Noise reduction compared to GRAPPA is more significant with higher accelerations.

Conclusions: We presented a new method for autocalibrating parallel MRI. We showed that coil sensitivity maps can be calculated using an eigen-value decomposition of the operators in acPI. These maps can be used in a SENSE reconstruction, with all the benefits of autocalibration, producing robust, and optimal reconstructions.

References: [1] Pruessmann et. al MRM 1999;42(5):952-62 [2] Griswold et. al MRM 2002;47(6): 1202-10 [3] Griswold et. al, MRM 2004;52(5):1118-26 [4] Lustig et. al, MRM 2010;64(2):457-71 [5] Lustig et. al, ISMRM '10 pp 2870 [6] Zhang et. al, ISMRM '10 pp 4907 [7] Lai et. al, ISMRM '10 pp 345 [8] Samsonov et. al, MRM 2004;52(6):1397-406 [9] Wang J. 1st Workshop on Parallel Imaging, 2001 pp.89

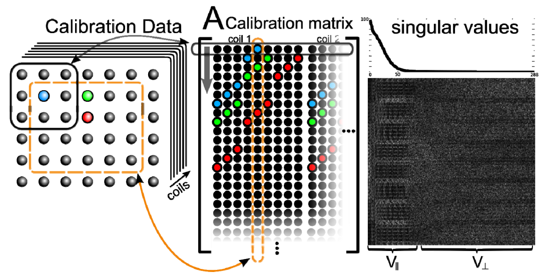

Figure 1: Calibration data is reordered into a calibration matrix, which is low-rank. Each overlapping block in k-space is spanned by $V_\parallel$

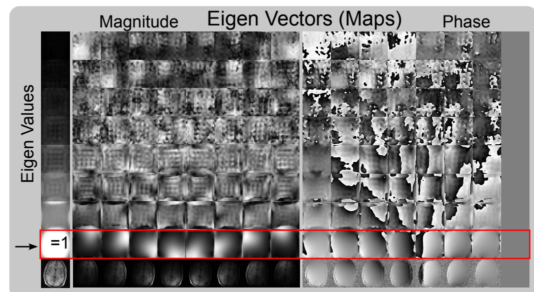

Figure 2: Eigen-vector maps of $V_\parallel V_\parallel^H$. The eigen-vectors with eigen-values '1' appear as very accurate sensitivity maps, both in magnitude and phase.

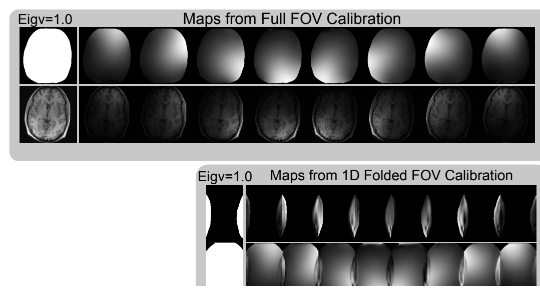

Figure 3: When the object is within the FOV, there is only one eigen-value '1' with a single set of maps. When the FOV is smaller than the object, in aliased regions there are multiple eigen-value '1'. Using both sets overcomes the FOV limitation described in [3].

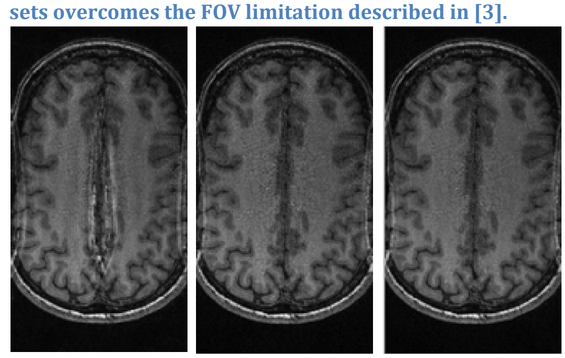

Figure 4: Reconstructions from 2-fold undersampling and prescribed FOV smaller than the object. Left: POCSENSE recon with mSENSE maps. middle: GRAPPA, right: POCSENSE recon with Eigen-vectors maps. Our reconstruction does not suffer from FOV limitation, and has less noise.