Modification of the simulated-multi-image method allows SNR measurement using sum-of-squares reconstruction

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Introduction

Signal-to-noise ratio (SNR) is a useful image metric, but evaluating the noise in an image can be challenging. A simple, reliable method ex vivo is to use multiple identical acquisitions to calculate the standard deviation of the signal in either an ROI or pixel-by-pixel, yielding an SNR map (the multi-image, or MI, method)[1]. However, due to subject motion, this approach is not viable in vivo. Recently, the simulated-multi-image SNR method (SMI method) was proposed[2], using the noise covariance matrix to appropriately weight the Gaussian noise added to each image in a simulated multi-image acquisition. The MI method is then applied to this stack of images to produce an SNR map. However, the added noise only propagates through the reconstruction accurately if the reconstruction can be expressed as a linear combination of matrix operators, so the SMI method is inaccurate for root-sum-of-squares (RSOS) reconstructed images. In addition, the magnitude reconstruction introduces noise bias, such that, at low SNR, the MI method overestimates the true SNR for RSOS images[3]. This work introduces two complementary methods which work in power space, the power-multi-image (PMI) and simulated-PMI (SPMI) methods, which correct for this noise bias and enable SNR measurement of in vivo RSOS images down to low SNR.

The power-MI and simulated-power-MI SNR methods

If RSOS-reconstructed images are squared to obtain a series of power images, $P_1 \ldots P_n$, the information contained in each pixel follows a χ^2 distribution. The mean and variance of the χ^2 distribution in each pixel are given by[3]:

$$\mu_{\chi^2} = 2n\sigma^2 + A_n{}^2 \quad \text{and} \quad \sigma^2_{\chi^2} = 4n\sigma^4 + 4\sigma^2 A_n^2,$$

where n is the number of receive channels, A_n the signal from each of the n channels and σ the standard deviation of the noise in each channel. These simultaneous equations can be solved for the underlying signal and noise:

$$A_n^2 = \sqrt{\mu^2_{\chi^2} - n\sigma^2_{\chi^2}} \quad \text{and} \quad \sigma^2 = \frac{\mu_{\chi^2} - A_n^2}{2n}.$$

The SNR is defined as A_n/σ, hence the PMI SNR is given by

$$\mathrm{SNR} = \sqrt{\frac{2n\alpha}{\mathrm{mean}(P_1^2 \ldots P_n^2) - \alpha}}, \text{ where } \alpha = \sqrt{\mathrm{mean}(P_1^2 \ldots P_n^2)^2 - n \cdot \mathrm{variance}(P_1 \ldots P_n)}.$$

This PMI method can be applied to the output of RSOS-reconstructed images which have had simulated noise added following the scheme in ref. 2. This is because the reconstruction of power images can be expressed as a linear combination of matrix operators. This produces the simulated PMI method (SPMI), shown in Fig.1. The original data without added noise is reconstructed and input into the PMI SNR method to provide a signal measurement.

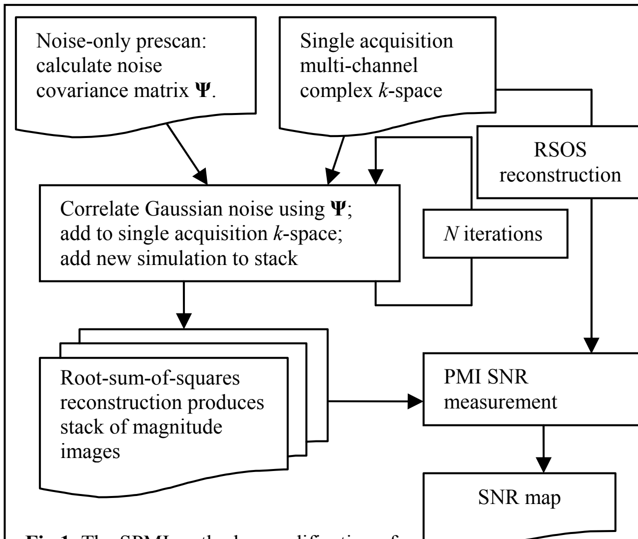

Fig 1. The SPMI method, a modification of the SMI method from ref. 2 applied to RSOS-reconstructed images using the PMI SNR method to process the simulated stack of images.

Methods

Multi-channel k-space data were acquired on whole-body GE (1.5T and 3T HDx, Waukesha, Milwaukee) and Siemens (3T Verio, Erlangen, Germany) systems. Multi-image data were acquired on phantoms, while single images were acquired on healthy volunteers under local ethics committee approval. The k-space-based simulations were carried out in MATLAB and images were either reconstructed using MATLAB (GE) or the standard 2D Siemens reconstruction. The SNR was analysed using the SPMI method in all cases, and the MI and PMI methods in phantoms.

Results & Discussion

The PMI method was found to agree to within 1% with the MI method on phantoms with values of SNR >15 in arrays with 8 or fewer elements. This is as expected given the bias introduced in the MI method at low SNR. The SPMI method was then compared to the MI method in phantoms, shown in Fig.2. Also included is the theoretical model from ref. 3 of the expected deviation of the MI method from the true SNR. The SPMI method does not agree perfectly with this model, due to non-diagonality of the noise covariance matrices of Arrays 3 and 4 breaking ref.3's assumption of a perfect χ^2 distribution. Nonetheless, the graph shows that the SPMI method provides a good estimate of the SNR even at noise levels when significant noise bias is introduced by magnitude reconstruction. Fig. 3 shows the application of the SPMI method to a RSOS reconstructed image of a healthy volunteer's brain, showing lower SNR on the left side of the image due to a faulty array element and zero SNR in the image background.

Conclusion

The power-multi-image (PMI) SNR measurement method enables accurate measurement of the SNR at high noise levels by correcting for noise bias. It can be applied to simulated multi-image (SMI) acquisitions for the measurement of SNR in vivo even at low SNR.

References

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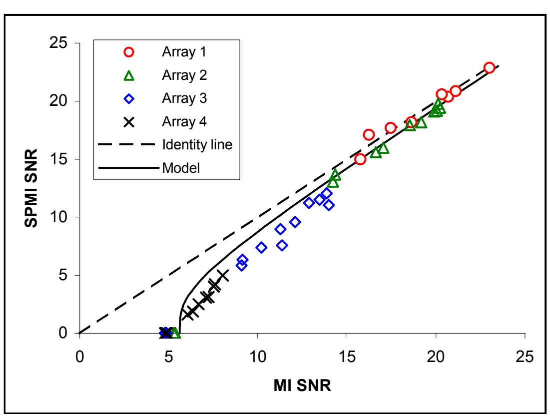

Fig 2. Comparing the MI and SPMI methods. Data are from ROIs on images of phantoms acquired with four different eight-channel brain arrays and 32 iterations. At high SNR there is good agreement between the MI and SPMI methods, while at lower SNR the MI method overestimates the SNR relative to that measured by the SPMI method, in reasonable agreement with the parameter-free model from ref. 3.

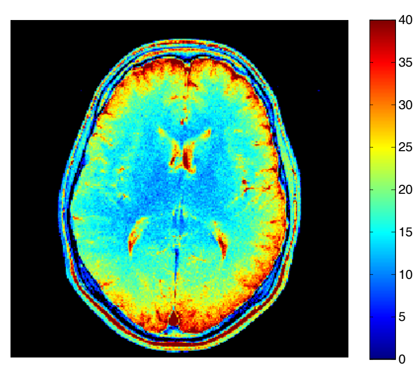

Fig 3. An SNR map of a normal volunteer's brain produced using the pseudo-multi-power-image method with 100 iterations.