Spatially variable Rician noise in DTI

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Introduction

During acquisition, MR images can appear to be corrupted by various artefacts due to noise. This problem becomes especially critical at low signal-to-noise ratios (SNR). In general, in modern fast imaging techniques using multiple-coils, local inhomogeneities of the magnetic field, bulk body motions, etc, give rise to the spatially-variable noise fields across an image [1]. Conventional noise correction schemes applied in diffusion tensor imaging (DTI) do not account for the spatial variability of the noise. Following the idea of Ref. [1], in this work we have developed a noise correction scheme which takes account of the spatial inhomogeneity of the noise, that is, the noise level is individually evaluated for each voxel. However, the procedure in Ref. [1] is valid only for high SNR under the assumption of Gaussian noise. In contrast, we used a Rician correction to the robust Gaussian estimator based on the median absolute deviation which is valid irrespective of the SNR level. The developed algorithm has been examined using numerical simulations with known spatial noise distributions and *in vivo* DTI experiments.

Theory and Methods

A standard deviation of the Gaussian noise distribution, σ_G, can be evaluated in each voxel using the MAD estimator: $\sigma_G = K \cdot MAD$, where coefficient K = 1.4826 [2]. For high SNR, the Gaussian and Rician noise estimations tend to coincide. However, in the voxels with low SNR, the initial estimation based on the Gaussian assumption has to be corrected for Rician noise. We perform a corresponding correction with the help of an analytical expression proposed in Ref. [3] for the Rician SD, $\sigma_R = \sqrt{\sigma_G^2 / \varsigma(\theta)}$, where function *ζ(θ)* is defined as $\varsigma(\theta) = 2 + \theta^2 - \frac{\pi}{8}\exp\left(-\frac{\theta^2}{2}\right)\left\{\left[2+\theta^2\right] I_0\left(\frac{\theta^2}{4}\right) + \theta^2 I_1\left(\frac{\theta^2}{4}\right)\right\}^2$. Here, $I_{0,1}(*)$ are zeroth/first order modified Bessel functions, respectively. The correction function *ζ(θ)* can be determined from a transcendent equation: $\theta = F(SNR),\ F(SNR) = \sqrt{\varsigma(\theta)\left[1 + \frac{\langle S\rangle^2}{\sigma_G^2}\right] - 2}$, where <*S*> is an averaged signal for the given voxel and *F*(SNR) is a function depending on the SNR defined as $\langle S\rangle/\sigma_R$. The image of the MR signal shown in Figure 1a was simulated numerically in such way that original signal was equal to 100 untis for a whole image but the SD was different in three different sub-regions. In the left upper quadrant, the noise was assumed to be Gaussian with SD equal to 5 units; in the left lower quadrant the noise assumed to be the Rician with SD equal to 20 units. The right half of image is distorted by Rician noise with SD equal to 45 units. *In vivo* diffusion brain experiments were carried out with a whole-body 3T Siemens MAGNETOM Trio scanner (Siemens Medical Systems, Erlangen, Germany). Diffusion weighted images were acquired for 15 diffusion weights in a range between 0 and 7000 s mm^{-2} and 6 non-collinear directions of the encoding diffusion gradients.

Results and Discussion

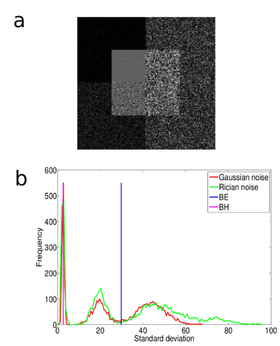

Figure 1

Simulated image and histograms of method comparison

We evaluated SD for the simulated image shown in Figure 1a using the developed approach and two classical methods for comparison. One of the latter is based on the background region estimation (BE) [1]. Another classical method utilizes the Brummer histogram algorithm (BH) [1]. The corresponding histograms are shown in Figure 1b for Gaussian noise, Rician noise, and the BH and BE methods. The developed spatial algorithms gave rise to three peaks (at about 5, 20 and 47 units) in the accordance with the simulated image. The classical algorithms, in contrast, produced an averaged, very rough estimation of SD. Figure 2 shows the experimental diffusion images for 4 different diffusion weightings. Two voxels in which the signal attenuates at the different rates were selected for analysis: the lower red arrow is associated with voxel 1 in which the signal attenuates relatively slowly with increasing diffusion weightings and, therefore, the residual SNR remains high; the upper red arrow points to voxel 2 in which the signal attenuates fast, and the remaining SNR at high diffusion weightings becomes low. We applied the Gaussian and Rician noise correction schemes to signal attenuations in these voxels. Classical BE and BH algorithms were also used for a comparison. Corrected data were used thereafter to reconstruct diffusion tensors associated with selected voxels. Mono- and bi-exponential diffusion tensors were fitted using a constrained non-linear least squares method [4]. Mono-exponential fits were restricted to the range of the diffusion weightings below 1000 s mm^{-2}; bi-exponential fits were performed in the full range of the acquired data. The evaluated data are presented in ellipsoidal form (main axes of the ellipsoids are related to the eigenvalues of the tensors) in Figure 3A (mono-exponential), 3B ("fast" tensor) and 3C ("slow" tensor). The differences observed for different noise correction methods were especially strong in the case of the "slow" tensor. These results will be discussed in the context of the residual noise level.

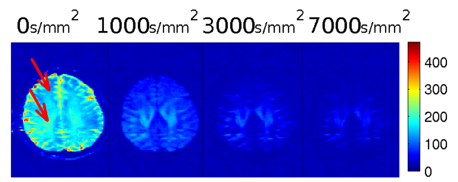

Figure 2

Diffusion weighted images

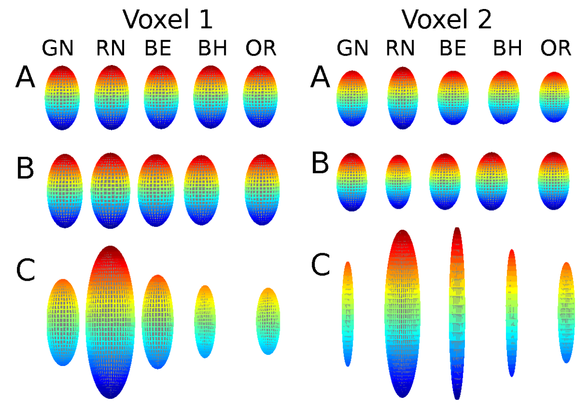

Figure 3

Comparison of different noise correction schemes

Conclusion

The proposed spatially-variable Rician noise correction algorithm was shown to be beneficial for the post-processing of DTI experiments due to its ability to treat noise at low SNR more precisely than using the Gaussian approach or the classical algorithms.

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References

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