A variational approach to susceptibility estimation that is insensitive to B0 inhomogeneity

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$$B = B_0\left(\frac{\chi}{3} - \left(\frac{\partial^2}{\partial z^2}\frac{1}{r} * \chi\right)\right) \quad [1]$$

$$\Box\chi = 3\,\Delta B / B_0 \quad [2]$$

$$\text{where} \quad \Box \doteq \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2}{\partial z^2}$$

$$\min_{\chi} \| F\left(\frac{\chi}{c_1}\right) - F\left(\frac{M}{c_2}\right) \|_2^2$$

$$s.t. \quad |W(\Delta B - \frac{B_0}{3}\Box\chi)|_1 < \epsilon \quad [3]$$

Introduction

In MRI, magnetic susceptibility differences cause measurable perturbations in the local magnetic field that can be modeled as the convolution of a dipole-like kernel with a spatial susceptibility distribution. In the Fourier domain, the kernel exhibits zeros at the *magic angle*, preventing direct inversion of the field map; also, limited observations make the problem ill-posed. The observed data is also corrupted by confounding fields (ie. those from tissue/air interfaces, mis-set shims, and other non-local sources). Previous work has shown that MR images can be successfully reconstructed from under-sampled observations by exploiting the sparsity of in-vivo data under various transforms using methods from compressed sensing [1]. In susceptibility estimation, the forward model results in under-sampling of the data in the Fourier domain, but accurate estimates can be obtained using the Laplacian and L_1 norm, which promote sparse solutions while removing external field artifacts.

Methods

We describe a variational method for susceptibility estimation that is based on the Laplacian operator. The solution of Maxwell's equations for the perturbing field, B, results in the forward model given in Eq. 1, where B_0 is the main field strength, χ, is the unknown susceptibility map, and $r = \sqrt{x^2 + y^2 + z^2}$. This equation consists of a local linear term and the convolution of χ with the second z-derivative of the $1/r$ distribution, which is the Green's function for the Laplacian [2]. Applying the Laplacian to the forward model results in Eq. 2., where $\Box$ is the D'Alembertian wave equation operator with two space dimensions, x and y, with z taking the place of "time", and speed, $c = 1/\sqrt{2}$. Eq. 2 shows that ΔB now depends only on local properties of χ . Also, since active shims produce fields that can be modeled with a spherical harmonic expansion, which is a solution of the Laplace equation, the effects of any mis-set shims and remote susceptibility distributions (ie. the neck/chest) are effectively eliminated by taking the Laplacian of the observed magnetic field. Method [3] has shown good results, but relies on sequential bias correction and susceptibility estimation (via filtering prior to solving for χ) and may not be able to recover from imperfections in bias field removal, while the method presented here is insensitive to these confounding fields. Since local susceptibility distributions can produce low-order fields that may appear to be due to sources outside the region of interest, some form of regularization is still needed in the form of regularizers [3], prior models, or atlases for a complete solution. Our method is defined in Eq.3, where M is the magnitude image, W is a mask of the region of interest, and c_1 and c_2 are constants. The first term penalizes solutions with large differences in spatial frequency structure relative to the magnitude data, and the second penalizes departures from Eq. 2, effectively, by enforcing agreement of high frequency phase effects while removing low order bias fields. Computation time is decreased by eliminating the calculation of Fourier transforms via the $\Box$ operator. The optimization is formulated using Lagrange multipliers and solved using conjugate gradient.

Data acquisition: For validation, two phantoms were constructed using Magnevist (gadopentetate dimeglumine) solutions of 0.5, 1.0, 2.0, and 3.0 mM corresponding to susceptibility values of 0.15, 0.31, 0.62, and 0.94 ppm respectively. Samples were placed in NMR tubes 10 cm in length and 3.43 mm inner diameter, and the molar susceptibility of 0.027 cgs units/mol Gd was used for conversion [4]. In the first experiment, the 2.0 mM sample was placed horizontally in a rectangular tank, transverse to the B0 field. Phase maps of the tube's center slice were obtained: 256x256 pixels, FOV=180mm, thickness=7mm, TR=100ms, TE=5.0, 7.25ms, and 10 averages [5]. In the second experiment, the tubes were separated by plastic disks and placed in a plastic cylinder 22.5 cm in length by 22 cm inner diameter and imaged using a 3D multi-echo GRE sequence: 128x128, 128 mm FOV, thickness=3 mm, TR=19ms, TE=6,12ms. Scanning was done on a 3T Siemens Trio MRI. B0 maps from both experiments were computed following phase unwrapping with Prelude [6].

Results

Application of the Laplacian removes the substantial inhomogeneity effects in both phantoms as seen in Fig.1. In the rectangular phantom, mean estimated susceptibility values for the water region and Gd tube were -9.049 and 0.6273 ppm with actual values of -9.05 and 0.6270 ppm. In the cylindrical phantom, the reconstructed susceptibility map allowed tubes that differed by Less than 1 ppm to be clearly identified and reasonable accuracy was obtained in the presence of significant noise and bias due to external field effects.

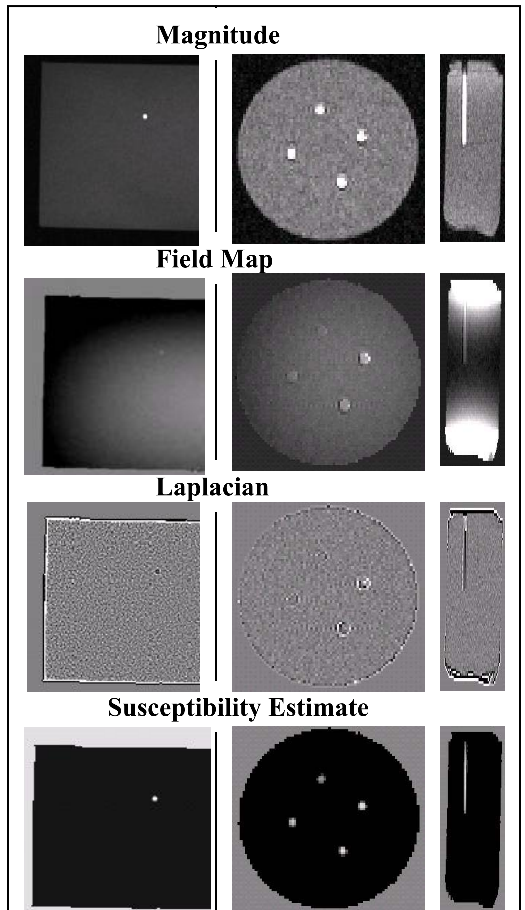

Fig. 1: Left column: rectangular phantom, right two columns: cylindrical phantom (see text for explanation).

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